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**KEUFFEL & ESSER CO.** MORRISTOWN, NEW JERSEY
BRANCHES AND DISTRIBUTOR OFFICES IN ALL PRINCIPAL CITIES

PRINTED IN U.S.A.

110998-81L-5

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# COMPENSATING POLAR PLANIMETERS

## INSTRUCTION MANUAL

**KEUFFEL & ESSER CO.**

# COMPENSATING POLAR PLANIMETERS

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**KEUFFEL & ESSER CO.**

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# COMPENSATING POLAR PLANIMETERS

## FIXED ARMS

### 62 0000

Compensating Polar Planimeter, reading areas directly in *square inches* to 0.01 sq. in. Can be used on maps and drawings to various scales. Particularly useful on maps with scale of 1 in. = 660 ft., giving direct readings in acres to 0.1 acre.

Length of tracer arm, $4\frac{1}{4}$ in.
Length of pole arm, $7\frac{1}{2}$ in.

### 62 0002

Same as No. 62 0000, but with tracer point in place of tracer lens.

### 62 0005

Similar to No. 62 0000, but reading areas directly in *square centimeters* to 0.1 sq. cm.

Length of tracer arm, $6\frac{3}{8}$ in. (16.2 cm.)
Length of pole arm, $7\frac{1}{2}$ in. (19.1 cm.)

### 62 0010

Similar to No. 62 0000, but for use on maps with scale of 1 in. = 330 ft., giving direct readings in *acres* to 0.1 acre. It can also be used for reading areas directly in square inches to 0.04 sq. in.

Length of tracer arm, $8\frac{3}{16}$ in.
Length of pole arm, $7\frac{1}{2}$ in.

## ADJUSTABLE ARMS

### 62 0015

PARAGON® Compensating Polar Planimeter, reading areas in *square inches* to 0.01 sq. in., or *square centimeters* to 0.04 sq. cm., or other units to scale.

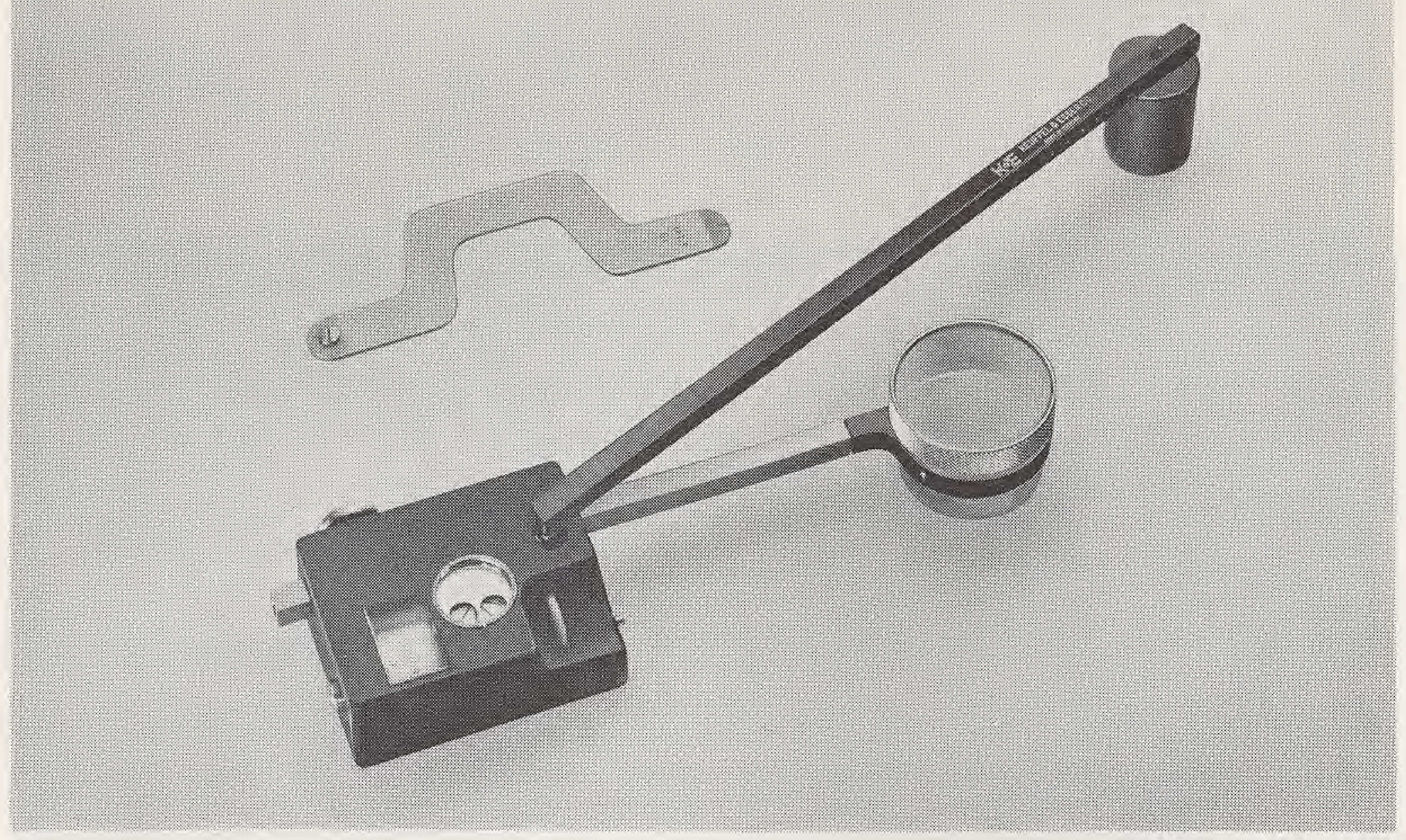
Range of tracer arm length, $2\frac{1}{2}$ to 7 in.
Range of pole arm length, 6 to 13 in.

### 62 0022

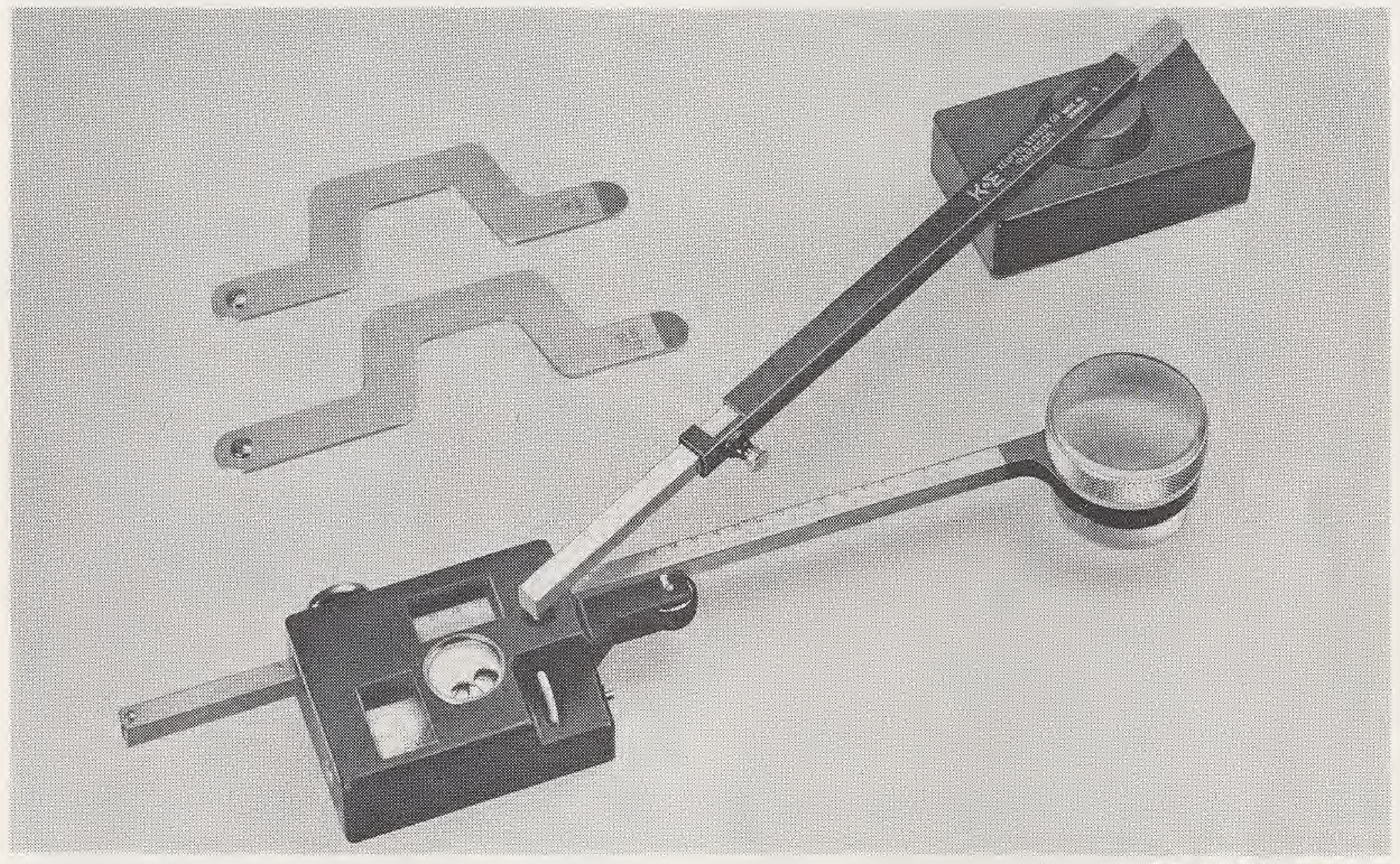
ANVIL® Compensating Polar Planimeter, performs same operations as No. 62 0015, but is an economy model.

*Note: For measurable areas of above instruments see page 7.*





No. 62 0000



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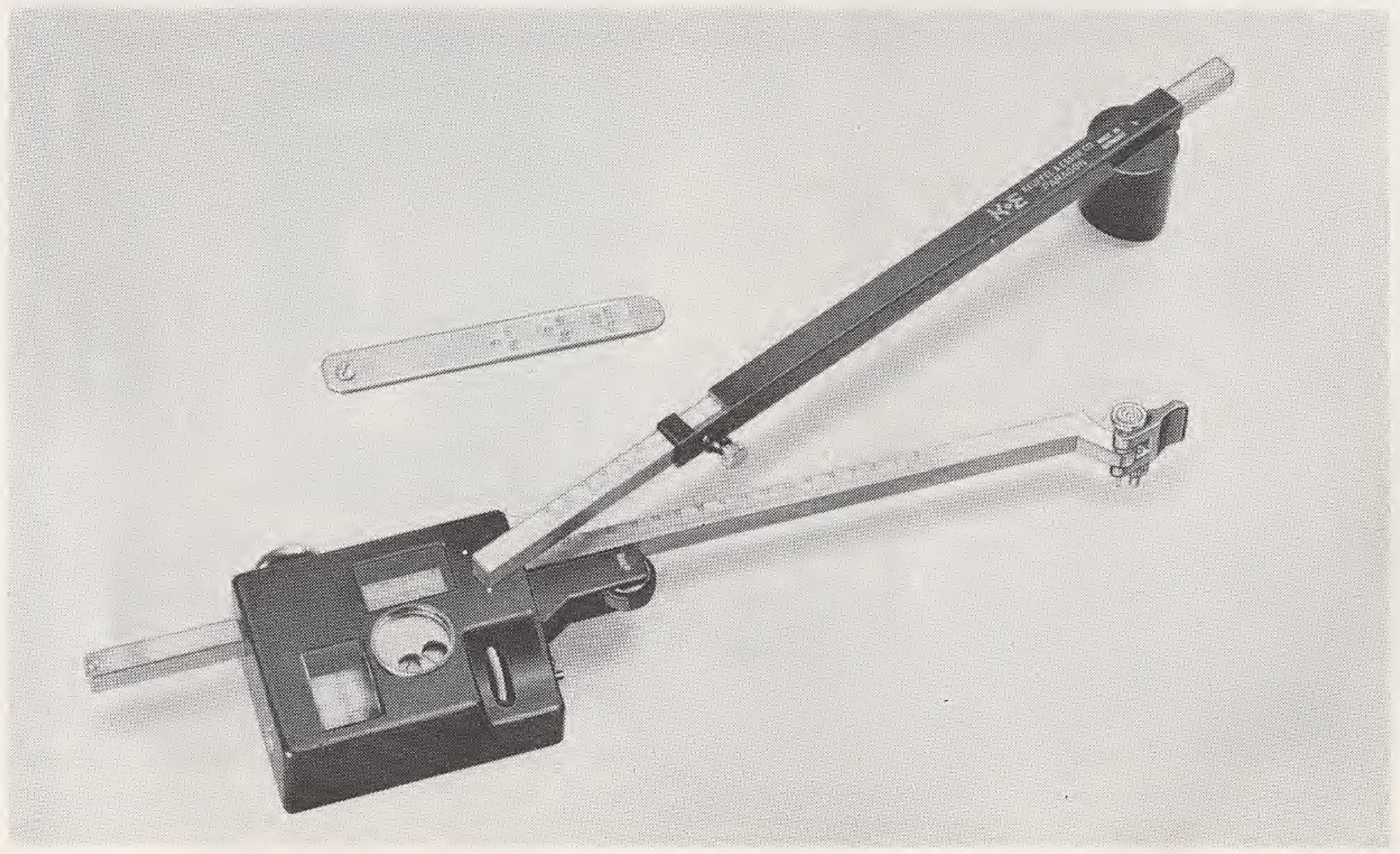


No. 62 0022

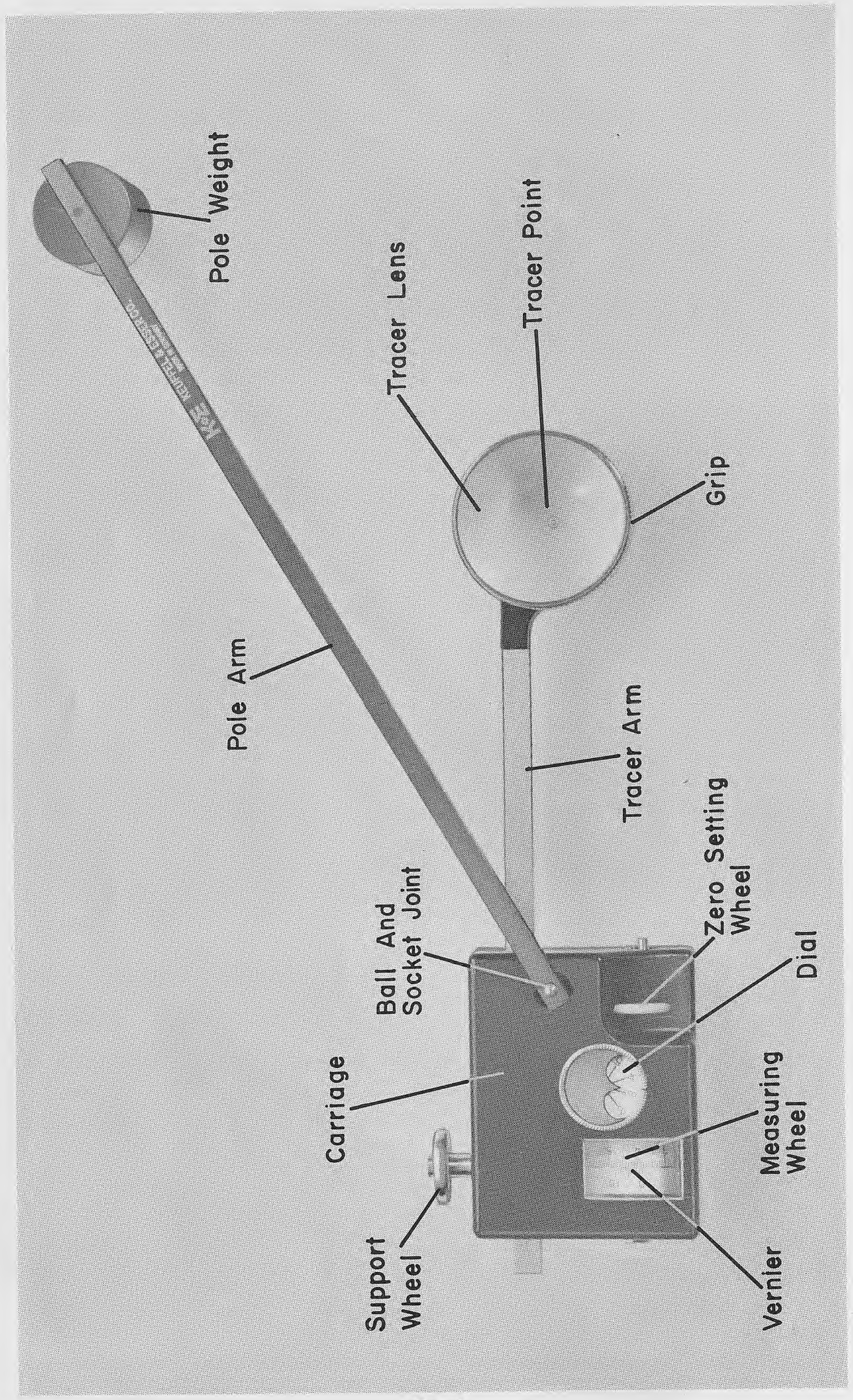



Fig. 1



## THE COMPENSATING POLAR PLANIMETER

These instructions are written on the assumption that the reader has little or no familiarity with the planimeter. The indulgence of the engineer is asked on this account, but he may find information of value, particularly in the latter part of the text.

### Tracer Points

Most K&E Planimeters are equipped with a new, crystal clear tracer lens as shown in Fig. 2. The lens magnifies the line being traced and permits greater accuracy. A small circle within a larger circle on the bottom of the lens is moved over the line being traced. Some K&E Planimeters have a tracer point and stop, as shown in Fig. 3. In the text which follows, reference to the tracer point can mean either type.

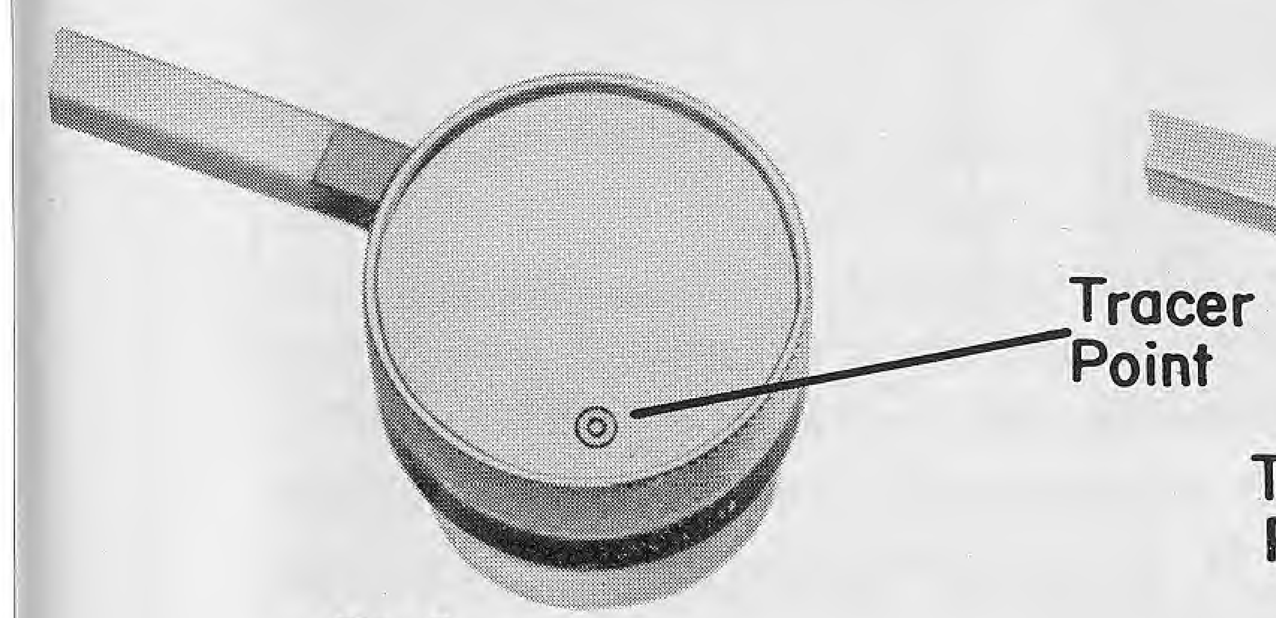



Fig. 2

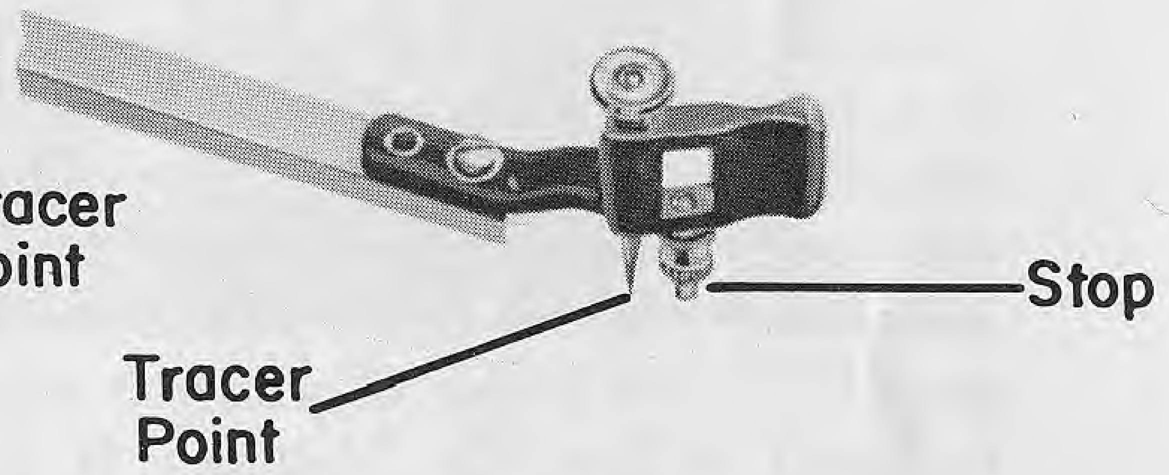



Fig. 3

### I. How the Planimeter Works

The Polar Planimeter is a simple instrument for the accurate measurement of plane areas of any form. To measure an area it is only necessary to run a tracer point around the periphery of the figure and read the distance which a measuring wheel has revolved during the process.

The planimeter consists of two units; the *pole arm* unit, and the *tracer arm* and *carriage* unit. The *pole arm* unit is simply a bar, at one end of which is the *pole*, consisting of a weight with a needle point at the bottom. At the other end is a *ball* which fits into a socket in the carriage. Now examine the *tracer arm* and *carriage* unit. Note that it rests on three points:—the measuring wheel rim, the support wheel and the tracer point.

Swing the carriage around the tracer point. Note that the measuring wheel turns rapidly, and the dial turns slowly. Now move it parallel to the tracer arm and note that the measuring wheel slides without turning. This characteristic sliding or turning of the measuring wheel is the key to the planimeter performance.

## II. Care

K&E Planimeters are built for accuracy and long service, but, like all precision instruments, they must be taken care of. If dropped, they will almost certainly be damaged. Do not therefore use a planimeter on a sloping board. Do not leave it lying around when drawings are being moved about. When not in actual use, it is best left in its case.

The most critical parts are the rim of the measuring wheel and the axle bearings. The wheel rim has a matte surface of hardened steel, which should be guarded against rust or corrosion. Do not touch it with the fingers. If it has been exposed to moisture, wipe it with a soft clean cloth. Be sure the surface, on which the planimeter is used, is even and free from grit or dust which could cause wear on the measuring wheel. If the planimeter is not to be used for some time, particularly in a humid climate, apply a coating of watch oil to the wheel rim, and wipe it off again before using the instrument. Keep the lens clean by occasionally wiping with a soft cloth or tissue.

Lubricate the axle bearings occasionally with a small drop of watch oil on a toothpick. Wipe off excess oil. The wheel should spin freely, but without perceptible end-play. Even after long use the bearings will not wear perceptibly. Do not attempt to adjust them. The accuracy of the instrument can always be checked with the test rule as explained on page 17. If any major adjustments are necessary, return the instrument to Keuffel & Esser Co., Hoboken, N. J., or if there is not time for this, an instrument specialist should attend to it.

## III. Operation

Place the carriage with the tracer arm parallel to the front edge of the board, the tracer point to the right and in the center of the area to be measured. Insert the pole arm ball in its socket in the carriage with the pole arm perpendicular to the tracer arm (See Fig. 13a). Press the pole needle firmly into the paper. If the pole weight anchor is used, insert the ball end into the socket of the anchor. This is the regular set-up of the planimeter.

On planimeters that have a tracer point and stop (Fig. 3), hold the tracer grip with thumb and second finger and push the tracer point down with the index finger. Adjust the stop so that the tracer point just clears the paper.

Holding the grip, move it about. Note that twisting the grip will lift the guide wheel off the paper. This should be avoided, and if it happens during a measurement, the measurement must be cancelled.

Test the limits of the motion of the tracer point from side to side and note that it can be moved freely whenever the tracer arm and pole arm make an angle within the wide arc between about 15° and 165°.



The shaded area in Fig. 4 represents the limit of measurable areas ("pole-outside-figure"). As an example the following table gives the limiting measurable areas of certain regular figures.

| *Planimeter Number* | *Greatest Square* | *Greatest Rectangle* | *Greatest Circle* |
|---|---|---|---|
| 62 0000 } 62 0002 } | 6¾ x 6¾ in. | 4 x 15 in. | 7 in. diam. |
| 62 0005 | 27 x 27 cm. | 20 x 50 cm. | 30 cm. diam. |
| 62 0010 | 12 x 12 in. | 10 x 19 in. | 13 in. diam. |
| 62 0015 } 62 0022 } | 11½ x 11½ in. | 6 x 29 in. | 12 in. diam. |

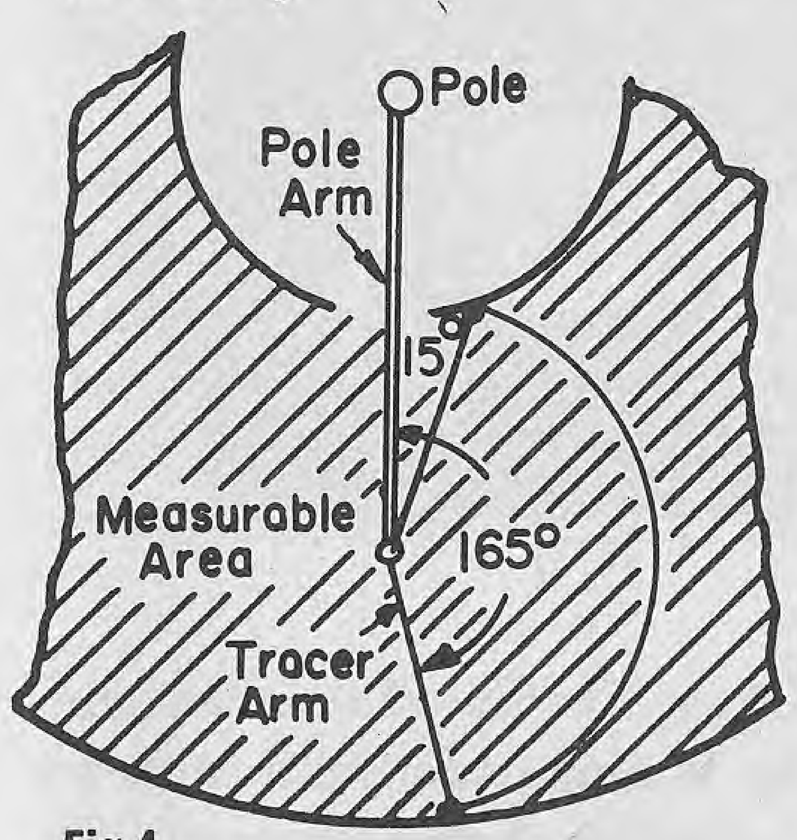



Fig.4

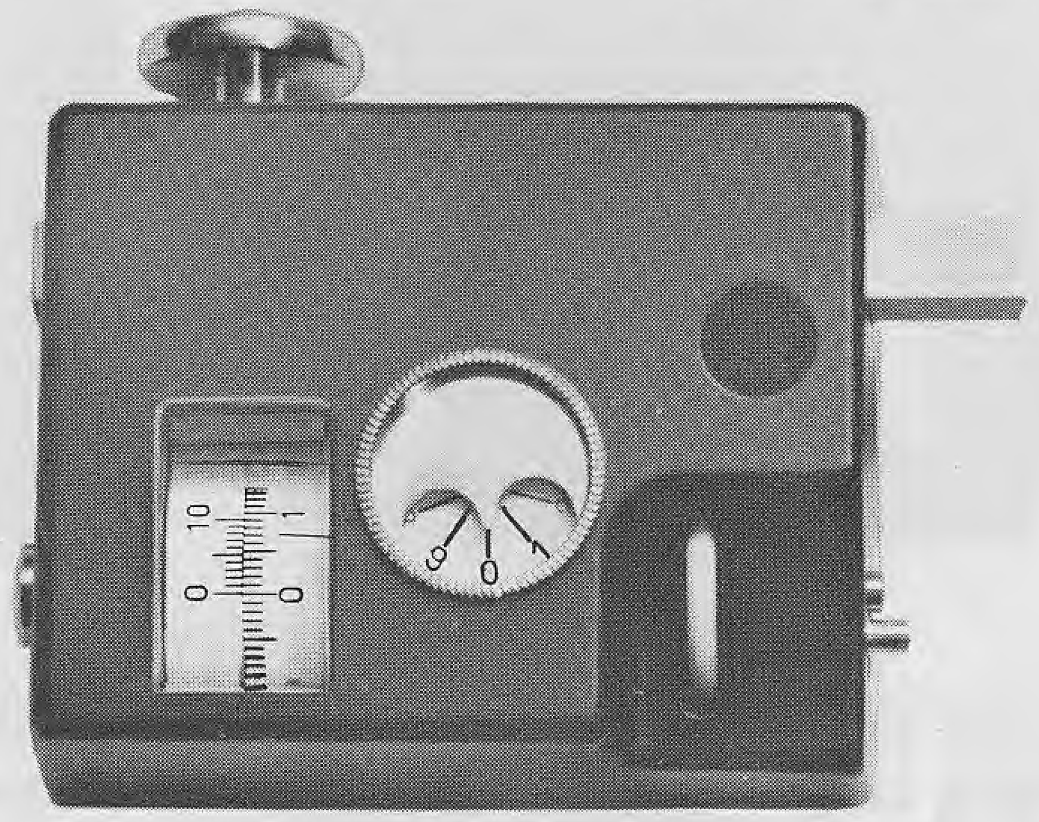


Fig. 5

**How to Make Readings** Move the tracer point in a series of clockwise circles and, watching the dial and the wheel, stop the motion when the pointer on the dial reads 0, and the 0 on the wheel is exactly opposite the 0 on the vernier. This is the zero position. (Fig. 5) Note that the 10 on the vernier is opposite the 9th graduation on the wheel, so that ten vernier graduations are opposite nine on the wheel. Now move the tracer point very slightly to the left or right (depending on the type of instrument), stopping before the vernier 0 has reached the first wheel graduation. You will see that one and only one graduation on the vernier coincides with a wheel graduation. If the 3rd vernier graduation thus coincides, you know that the wheel has turned 3/10 the distance from its 0 to its first graduation. If it is the 4th, it has turned 4/10 the distance, etc. (Fig. 6) Thus the vernier is used just to tell you more accurately than you can judge by eye how to divide one division on the wheel into 10 equal parts.

Now move the tracer a little further to the left, or right, until the 0 on the vernier is opposite the first graduation on the wheel. The wheel has now moved 10 "vernier units" or 1 "wheel unit." (Fig. 7) When the wheel has turned all the way until the graduation numbered "1" on the wheel is opposite the vernier 0, it has moved 10 "wheel units" or 100 "vernier units." When the wheel has made a complete revolution past 2, 3, 4, etc. and on to the point where the two 0's again coincide,

it has moved 100 wheel units or 1000 vernier units. Notice that the dial has now moved so that the pointer is at "1" instead of at 0 (Fig. 8). Each of the 10 numbers on the dial corresponds to one complete revolution of the wheel. When the dial makes one complete revolution, the wheel makes 10 revolutions, or 10,000 vernier units. Thus, in terms of vernier units we read thousands on the dial, hundreds and tens on the wheel, and units on the vernier.

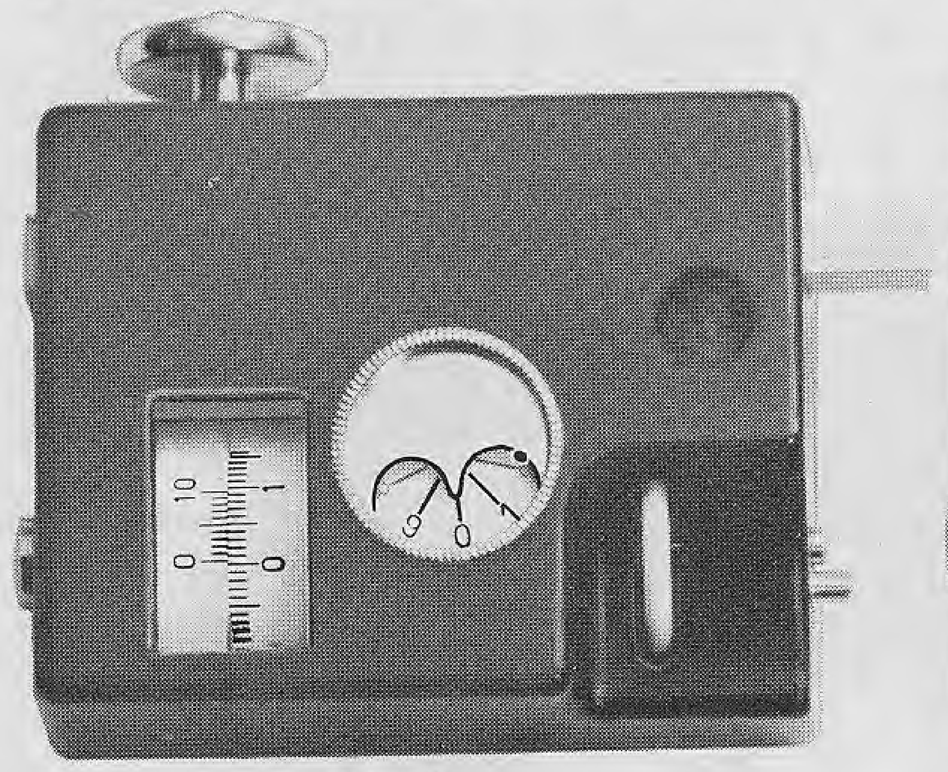


Fig. 6

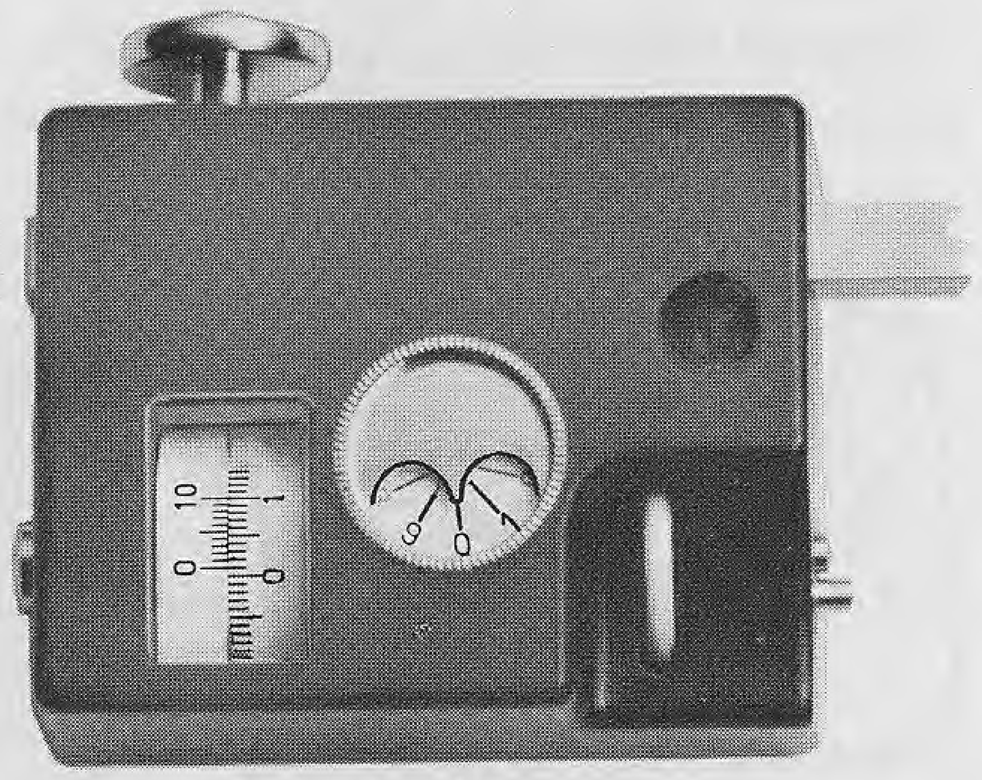


Fig 7

If the reading in wheel units is in the 80s or 90s, the dial index will be close to the next larger number and it is important not to read the dial one number too high. Thus, if the wheel and vernier read 927 and the dial index is near 6, the correct reading is 5927, not 6927.

As an example, in the illustration (Fig. 9), since the index on the dial is between 3 and 4, we know the reading in vernier units is 3000+; since the vernier 0 is between 4 and 5 on the wheel we know the reading is 3400+ vernier units; since the vernier 0 is between the 7th and 8th graduation (of the graduations between 4 and 5 on the wheel) we know the reading is 3470+ vernier units; finally, since the 4th graduation on the vernier coincides with a wheel graduation, the complete reading is 3474.

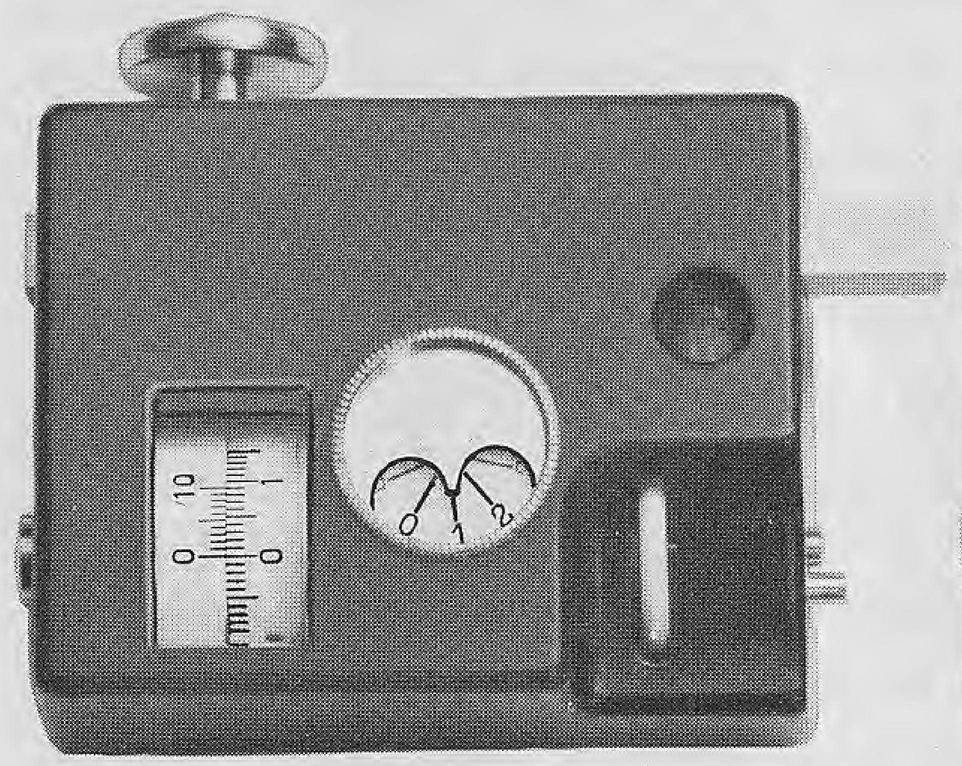


Fig. 8

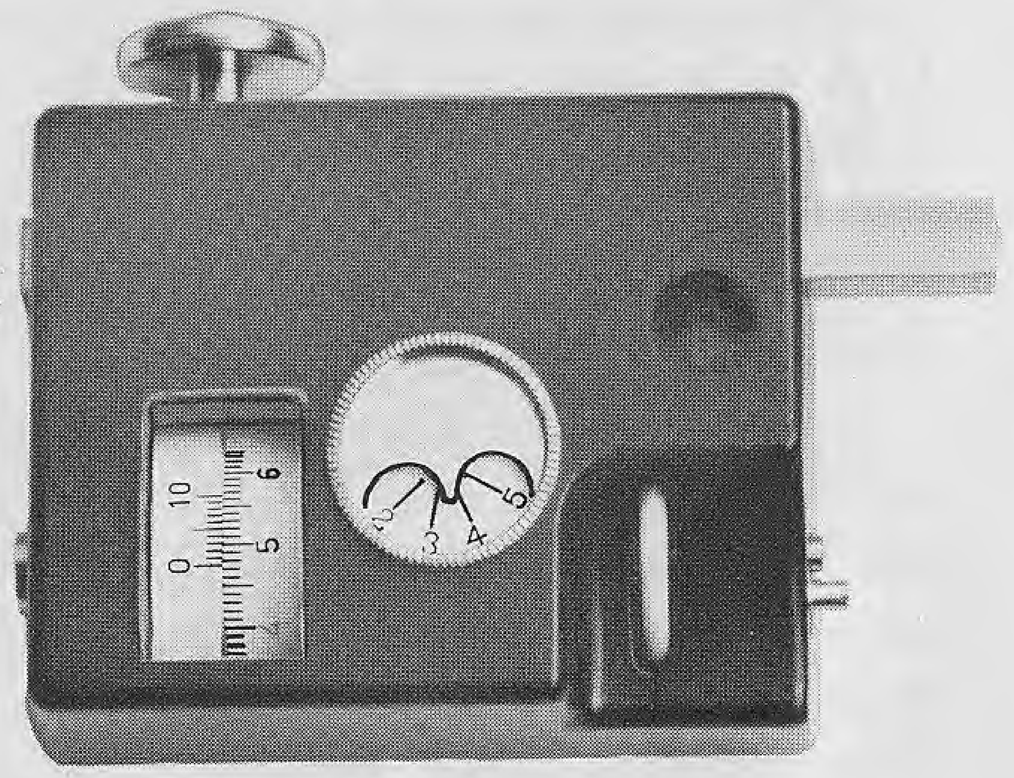


Fig. 9



This is the procedure by means of which any planimeter setting is read to four places. With a little practice the reading can be made very quickly and without error.

**The Value of a Vernier Unit*** K&E No. 62 0000† Polar Planimeter has a tracer arm of fixed length and it reads directly in square inches to hundredths. That is, 1 vernier unit = 0.01 square inches.

K&E No. 62 0005 Polar Planimeter has a fixed tracer arm and it reads directly in square centimeters to tenths. That is, 1 vernier unit = 0.1 square centimeters.

| | | No. 62 0000† | No. 62 0005 |
|---|---|---|---|
| One unit on vernier | = | 0.01 sq. in. | 0.1 sq. cm. |
| Single division on wheel | = | 0.10 " " | 1.0 " " |
| Major divisions on wheel | = | 1.00 " " | 10.0 " " |
| One revolution of measuring wheel | = | 10.00 " " | 100.0 " " |
| One division on the dial | = | 10.00 " " | 100.0 " " |
| One complete revolution of the dial | = | 100.00 " " | 1000.0 " " |

**Preparation for Tracing an Area** Assuming that the area to be measured is within the limits discussed on page 7, the pole can be placed in any position which will permit the tracer point to reach the entire periphery of the figure. For large areas see page 14.

The following precautions should be observed:

1. Set the pole in such a position that the angle between the tracer arm and the pole arm will always be between 15° and 165°. It is better, where possible, to keep the angle between 30° and 150°.
2. The surface of the paper, with which the wheel rim comes in contact during measurements, must be smooth, clean and uniform. If it is impossible to keep the wheel from moving off the paper, a smooth butt-joint must be made with paper of equal thickness, and the instrument positioned so the measuring wheel will roll over the joint rather than slide over it.
3. Draw a fine line across the periphery of the area to be measured to mark the starting and finishing point.
4. When the tracer point is nearest the pole, so that the tracer arm and pole arm make a sharp angle, it will be seen that the wheel revolves very rapidly. The tracer point should be moved very slowly over this part of the figure.

**Tracing An Area** Having made a rough preliminary circuit of the area, and established a starting (and finishing) point, you are ready to make a measurement.

---

**See page 12 for No. 62 0010.*
*†Also applies to 62 0002.*

1. With the tracer point precisely at the beginning line, take a reading and set it down. Assume the reading is 3476 vernier units.
2. Holding the tracer grip, follow the outline of the figure carefully in a clockwise direction.
3. If the tracer point inadvertently leaves the line it is not necessary to go back, but instead the tracer point may be intentionally moved the other way to compensate for the error. (Fig. 10)

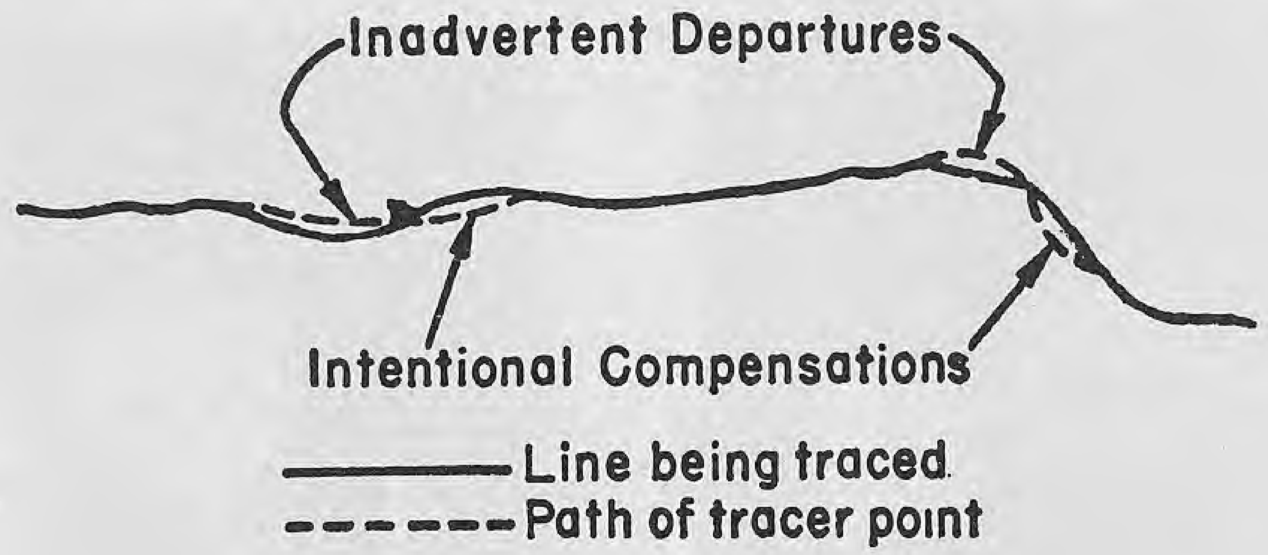



Fig. 10

4. Do not attempt to use a guide for straight lines, as this is apt to produce a cumulative error.
5. When the figure has been circumscribed so that the tracer has returned precisely to the starting point, make a second reading. Assume it is now 7491 vernier units.
6. Place this number above the first reading and subtract the first from the second:

| | |
|---|---|
| Second reading | 7491 |
| First reading | 3476 |
| Difference | 4015 |

7. The difference 4015 indicates the area. If the instrument used was K&E No. 62 0000, in which 1 vernier unit = 0.01 sq. in., then the indicated area is 4015 x 0.01 sq. in. = 40.15 sq. in. If the instrument was K&E No. 62 0005, in which 1 vernier unit = 0.1 sq. cm., then the indicated area is 4015 x 0.1 = 401.5 sq. cm. If No. 62 0015 was used, the indicated area depends on the tracer arm setting. See page 21.
8. It is best to repeat the tracing at least once to serve as a check. Additional tracings may be made and averaged to increase the accuracy of the results. See page 18 for a discussion of this subject.
9. If the 0 of the dial passes the index during the tracing, add 10,000 vernier units to the second reading. Thus:

| | | | |
|---|---|---|---|
| Second reading | 2765 | = | 12765 |
| First reading | 7452 | = | 7452 |
| Area | ? | | 5313 |



NOTE: The new Zero Setting Wheel on K&E Planimeters makes it possible to set the measuring wheel to 0 before the first tracing when only single tracings are made. For repeated tracings, the measuring wheel should not be set to 0 before each successive tracing so as to assure highest accuracy.

**Areas to Scale** Although No. 62 0000 and No. 62 0005 always measure areas in sq. in. and sq. cm. respectively, they may be used very easily to give areas to any scale, by simply multiplying the measured area by a *factor*. The slide rule is ideal for such work as it can be left set at the factor, and the conversion read immediately for each separate measurement.

The factor for any map or drawing to scale is easily determined. *It is the value represented on the map or drawing of 1 sq. in. for No. 62 0000 or 1 sq. cm. for No. 62 0005.* For example, suppose the map has a scale of 1 in. = 50 ft. The value of 1 sq. in. on the map is 50 × 50 = 2500 sq. ft. The factor is 2500. If the area measured was 13.76 sq. in. then the area on the map would be 13.76 × 2500 = 34,400 sq. ft. In the same manner, if No. 62 0005 were used on a map scaled 1:5000 and the result is required in sq. meters, the scale may be written 1 cm. = 50 m and 1 sq. cm. would represent 50 × 50 = 2500 sq. m. or a factor of 2500. Then if the measured area was 137.6 sq. cm. the area represented would be 137.6 × 2500 = 344,000 sq. m.

Taking several readings makes for greater accuracy. If the sum total is used, the factor must be changed accordingly. If repeated readings give a power of 10, the computation is simplified. For example, on a scale of 1 in. = 200 ft., if the area is traced 4 times, the factor becomes 200 × 200 ÷ 4 = 10,000. Thus if tracing the area four times gives a total reading of 7.62 sq. in., the indicated area is 7.61 × 10,000 = 76,200 sq. ft.

It is also a simple matter to convert the planimeter reading directly into special units of area, such as acres. For example, if the scale is 1 in. = 100 ft. and there are 43,560 sq. ft. in 1 acre, the area of 1 sq. in. on the map in acres is 100 × 100 ÷ 43,560 = 0.2296 acres. Thus for a reading of 21.05 sq. in. the acreage is computed as follows:

21.05 sq. in. × 0.2296 acres per sq. in. = 4.83 acres.

The same method can be used to correct errors due to distortion of the paper or cloth. If the nominal scale is 1 in. = 200 ft., but the paper has stretched 0.6% in the east-west direction and shrunk 0.2% in the north-south direction, 1 sq. in. on the map represents (200 ÷ 1.006) (200 ÷ 0.998) = 198.8 × 200.4 = 39,840 sq. ft. = 0.915 acres. Using either of these factors with planimeter readings will automatically correct errors due to distortion of the map.

For the convenience of the user, factors for commonly used architectural and map scales are given on pages 12 and 13.

## Table I

### FACTORS FOR SCALE DRAWINGS OR MAPS

### No. 62 0000, No. 62 0002, No. 62 0015 or No. 62 0022—English System.

FACTORS

*Factors by which sq. in. values must be multiplied to convert to units shown at column headings.*

| *Nominal Scale of Dwg. or Map* | *Actual Scale Ratio* | *sq. in.* | *sq. ft.* | *acres* | *sq. miles* |
|---|---|---|---|---|---|
| Double size | 2:1 | 0.25 | — | — | — |
| 1½ size | 1½:1 | 0.444 | — | — | — |
| Full size | 1:1 | 1.0 | — | — | — |
| Centimeters | 1:1 | 6.45* | — | — | — |
| Half size | 1:2 | 4.0 | — | — | — |
| Quarter size | 1:4 | 16. | — | — | — |
| 3 in. = 1 ft. | 1:4 | 16. | 0.111 | — | — |
| 1½ in. = 1 ft. | 1:8 | 64. | 0.444 | — | — |
| 1 in. = 1 ft. | 1:12 | 144. | 1.0 | — | — |
| ¾ in. = 1 ft. | 1:16 | 256. | 1.778 | — | — |
| ½ in. = 1 ft. | 1:24 | 576. | 4.0 | — | — |
| ⅜ in. = 1 ft. | 1:32 | 1024. | 7.11 | — | — |
| ¼ in. = 1 ft. | 1:48 | 2304. | 16.0 | — | — |
| ⅛ in. = 1 ft. | 1:96 | 9216. | 64.0 | — | — |
| 1 in. = 10 ft. | 1:120 | — | 100. | — | — |
| 1 in. = 20 ft. | 1:240 | — | 400. | — | — |
| 1 in. = 40 ft. | 1:480 | — | 1600. | 0.0367 | — |
| 1 in. = 50 ft. | 1:600 | — | 2500. | 0.0574 | — |
| 1 in. = 66 ft. | 1:792 | — | 4356. | 0.1 | — |
| 1 in. = 80 ft. | 1:960 | — | 6400. | 0.147 | — |
| 1 in. = 100 ft. | 1:1200 | — | 10,000. | 0.2296 | — |
| 1 in. = 132 ft. | 1:1584 | — | 17,424. | 0.4 | — |
| 1 in. = 200 ft. | 1:2400 | — | 40,000. | 0.918 | — |
| 1 in. = 330 ft. | 1:3960 | — | 108,900. | 2.5 | — |
| 1 in. = 400 ft. | 1:4800 | — | 160,000. | 3.673 | — |
| 1 in. = 660 ft. | 1:7920 | — | 435,600. | 10.0 | 0.0156 |
| 1 in. = ¼ mi. | 1:15,840 | — | — | 40.0 | 0.0625 |
| 1 in. = ½ mi. | 1:31,680 | — | — | 160.0 | 0.25 |
| 1 in. = 1 mi. | 1:63,360 | — | — | 640.0 | 1.0 |
| — | 1:25,000 | — | — | 99.6 | 0.1557 |
| — | 1:62,500 | — | — | 622.7 | 0.973 |

1/16 = 1'-0" 1:192 256

*This factor is square centimeters for reading areas in the metric system.

1/32 = 1'-0" 1:384

### No. 62 0010—English System

No. 62 0010 is a fixed arm planimeter for use on maps with scale of 1 in. = 330 ft. It gives direct readings in acres to 0.1 acres. (Reading x 0.1 = area in acres). It can also be used for measuring areas in square inches to 0.04 sq. in. (Reading x 0.04 = area in square inches).



## Table II

### FACTORS FOR SCALE DRAWINGS OR MAPS

### No. 62 0005, No. 62 0015 or No. 62 0022—Metric System.

FACTORS

*Factors by which sq. cm. values must be multiplied to convert to units shown at column headings.*

| *Scale Ratio of Dwg. or Map* | *sq. mm.* | *sq. cm.* | *sq. m.* | *hectares* | *sq. km.* |
|---|---|---|---|---|---|
| 10:1 | 1. | 0.01 | — | — | — |
| 5:1 | 4. | 0.04 | — | — | — |
| 2:1 | 25. | 0.25 | — | — | — |
| 1½:1 | 44.4 | 0.444 | — | — | — |
| 1:1 | 100. | 1.0 | — | — | — |
| 1:1 | — | 0.155* | — | — | — |
| 1:2 | 400. | 4.0 | — | — | — |
| 1:5 | 2500. | 25.0 | — | — | — |
| 1:10 | 10,000. | 100.0 | 0.01 | — | — |
| 1:100 | — | 10,000. | 1.0 | — | — |
| 1:1000 | — | — | 100.0 | 0.01 | — |
| 1:10,000 | — | — | 10,000. | 1.0 | 0.01 |
| 1:100,000 | — | — | — | 100.0 | 1.0 |

*This factor is sq. in. for reading areas in the English system.

**Indicator Diagrams** The mean effective pressure (MEP) indicated by the height of indicator diagrams (Fig. 11) can be easily obtained with No. 62 0000 or No. 62 0005. (See page 26 for No. 62 0015.)

1. Establish a factor by dividing the pressure scale by the length of the stroke on the card. 2. Measure the area of the diagram. 3. Multiply the area by the factor to obtain the MEP.

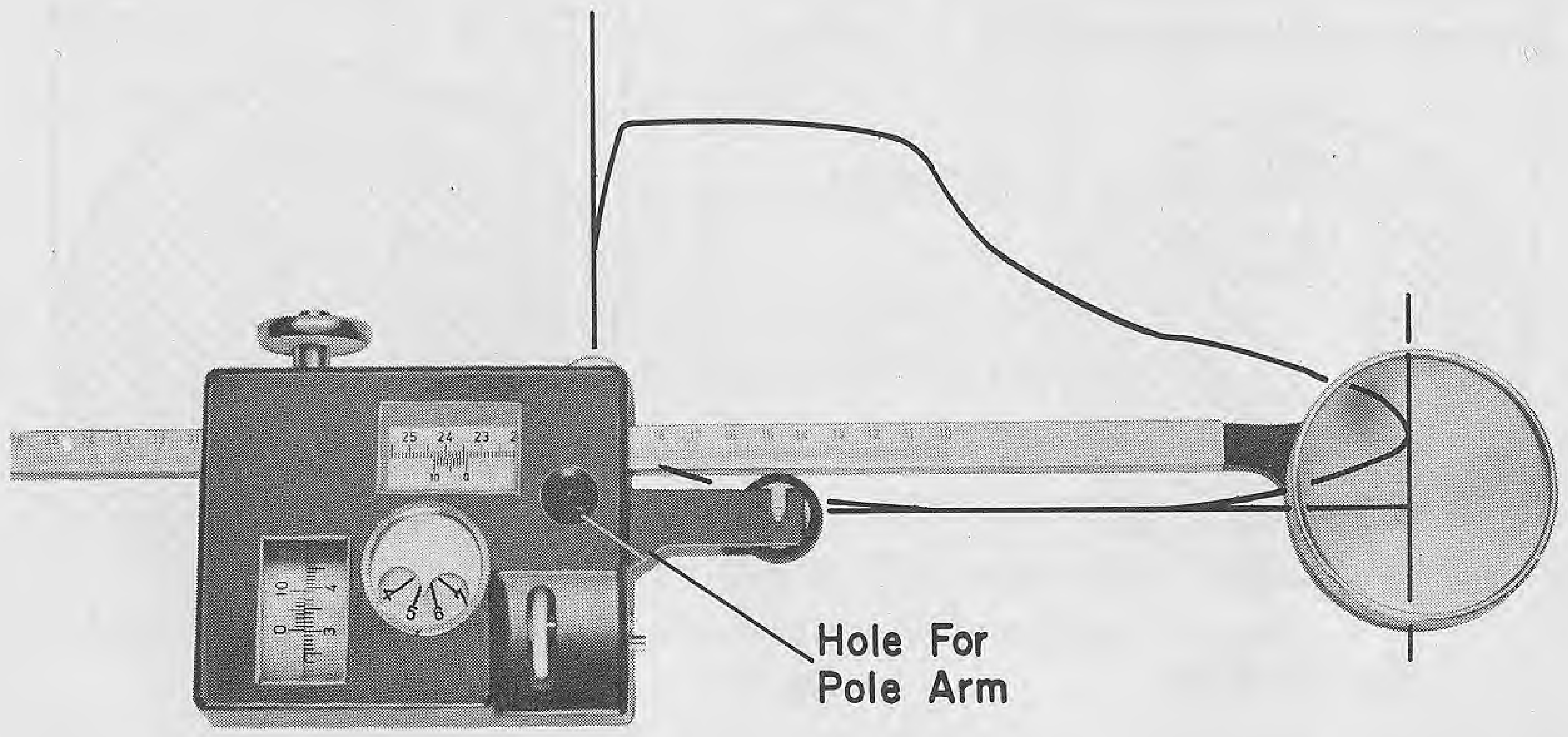



Fig. 11

For example, if the pressure scale is 1 inch = 80 lbs./in.$^2$ and the stroke on the diagram 4 inches, the factor would be $80 \div 4 = 20$. If the diagram has an area of 2.31 sq. in., the MEP is $2.31 \times 20 = 46.2$ lbs./in.$^2$

The planimeters may also be used with circular charts to find the mean radius of the trace, unless the trace has great eccentricity.* Measure the Area $A$ and find the radius $r$ from the formula

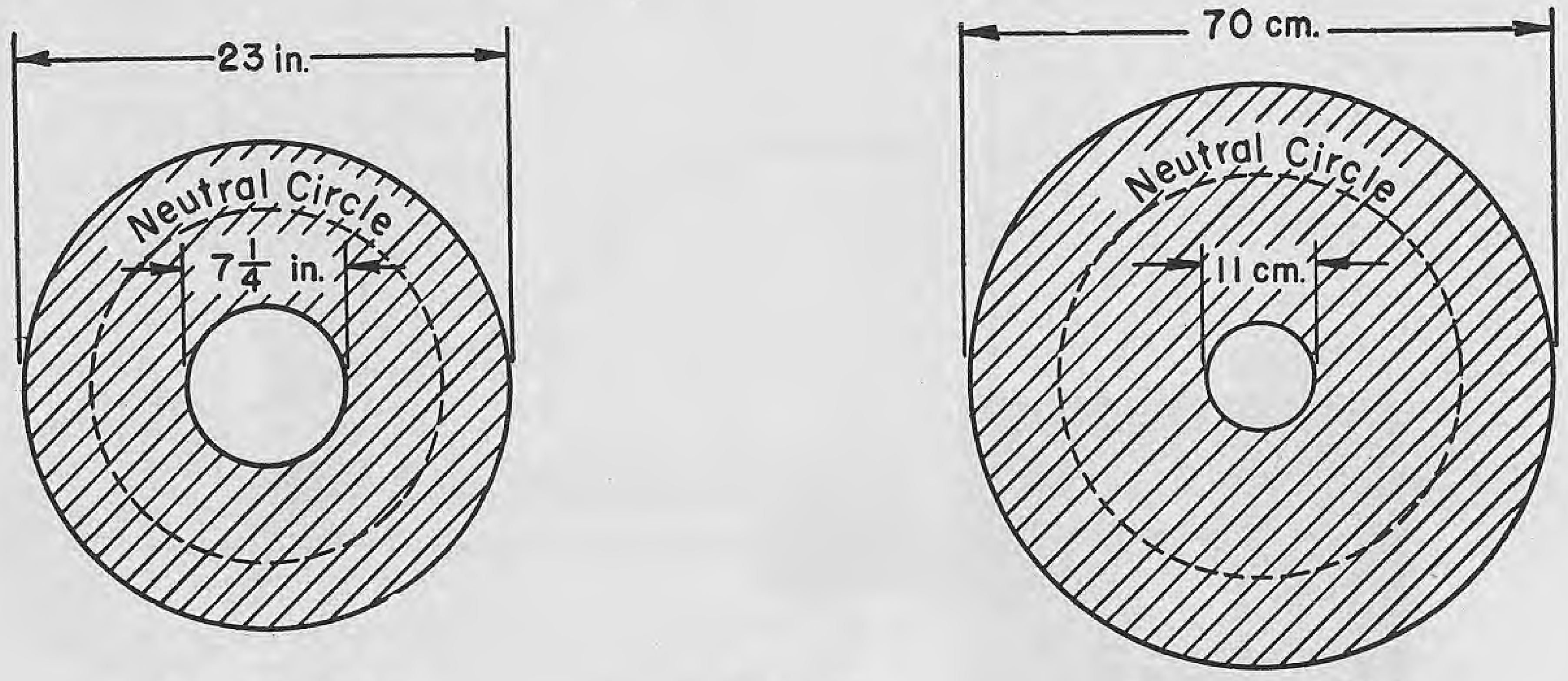
$$r=\sqrt{\frac{A}{\pi}} = 0.5642\sqrt{A}$$

On the circular chart, at a distance $r$ inches from the center, is found the mean value of the pressure, temperature, or whatever unit is being measured.

**Measuring Large Areas** When the area to be measured is larger than that described on page 7, two optional methods are available.

The first is the simple expedient of dividing the area into measurable segments, measuring each separately and adding the results. This method is recommended when no more than two or three segments are required. For larger areas the second method, known as the "pole-within-figure," is described below.

It is possible to measure a large area accurately by placing the pole near the center of the area and making a complete circuit around it with the tracer. Obviously, the area is limited to the periphery that can be reached with the tracer in this process. Thus the entire periphery of the figure must be of such size and shape that it can be made to lie within concentric minimum and maximum circles as indicated in the shaded areas of Fig. 12. Thus if the figure is no more than 23 in. across and is at least 7¼ in. across near the middle, it is measurable with No. 62 0000 in a single setting.




No. 62 0000 **Fig. 12** No. 62 0005

*The K&E 62 0100 Radial Planimeter is better suited for this work, as it is faster and more accurate. It can be used irrespective of the contour.*



The area of such a figure is measured by setting the pole near its center, and running the tracer in a clockwise direction around the periphery to the starting point. However, the reading thus obtained *does not indicate the area directly*, but instead it indicates the difference in area between that of the figure and that of the neutral circle. A simple computation gives the desired area as explained below. The neutral circle varies with each individual instrument and is shown inside the lid of the case. The neutral circle is the area circumscribed by the tracer point when it makes a complete revolution around the pole, at a uniform distance from the pole so that the wheel motion is all sliding. The theory is explained on page 30.

On all "pole-within-figure" measurements it is desirable to start with the dial and wheel at 0. This is done by holding the tracer firmly on the starting point with the right hand to prevent movement. Tilting the tracer point slightly will lift the measuring wheel from the paper to permit turning the zero setting wheel to 0. Do not turn the measuring wheel while it is in contact with the paper, or touch the wheel rim at any time.

Before making the measurement it is necessary to know whether the area is greater or less than that of the neutral circle. This is determined by noting whether the net motion of the wheel is plus or minus. Set the dial and wheel at 0 and make a quick approximate clockwise circuit of the area, watching the dial and noting whether its total motion has been positive or negative. It may make even more than one complete revolution, and if this occurs, the fact should be duly noted.

Thus, one of four possible conditions will be found. From the largest area to the smallest these conditions are: The net dial motion may be (1) positive and more than one complete revolution; (2) positive and less than one revolution; (3) negative and less than one revolution; (4) negative and more than one revolution. The formulas to use for each of these four cases are given below.

If $A$ is the area of the figure in vernier units, $C$ the area of the neutral circle in v.u., and $W$ the final reading of dial and wheel in v.u. after tracing the area beginning at 0000,

Case 1. $A = C + 10{,}000 \text{ v.u.} + W$
Case 2. $A = C + W$
Case 3. $A = C - 10{,}000 \text{ v.u.} + W$*
Case 4. $A = C - 20{,}000 \text{ v.u.} + W$*

The formulas are given in vernier units so they will apply to both No. 62 0000 and No. 62 0005. For No. 62 0000, 1 sq. in. = 100 v.u. and for No. 62 0005, 1 sq. cm. = 10 v.u. For scale drawings the factors in Tables I and II may be applied after $A$ has been found.

*The negative motion of the dial and wheel must be subtracted from C. The negative motion when read positively is (10,000 — W). Hence:*

$A = C - (10{,}000 - W) = C - 10{,}000 + W$ or

$A = C - 10{,}000 - (10{,}000 - W) = C - 20{,}000 + W$.

*Example of Case 1.* Using a No. 62 0000 planimeter having a neutral circle area of 216.91 sq. in., on a full size drawing, the dial was seen to move in a positive direction one complete revolution and about 4 dial units over, from a preliminary circuit of the area. When the area was carefully traced, the reading was 43.65 sq. in. $A = C + 10{,}000$ v.u. $+ W$.

$$\begin{aligned} C + 100.00 \text{ sq. in.} &= 316.91 \\ W &= \phantom{0}43.65 \\ \hline A &= 360.56 \text{ sq. in.} \end{aligned}$$

*Example of Case 2.* Using No. 62 0005 planimeter having a neutral circle area 1845.0 sq. cm. on a drawing to a scale of 1:50, a preliminary circuit showed a positive motion of about 7 dial units. The final traced reading was 685.2 sq. cm. $A = C + W$.

$$\begin{aligned} C &= 1845.0 \\ W &= \phantom{0}685.2 \\ \hline A &= 2530.2 \text{ sq. cm.} \end{aligned}$$

The factor is $50 \times 50 = 2500$ sq. cm. $= 0.25$ sq. m. Area to scale is $2530.2 \times 0.25 = 632.55$ sq. m.

*Example of Case 3.* With the instrument and drawing of Example 1, the preliminary circuit showed the dial moving backward about six dial units. The reading after tracing was 41.64 sq. in. $A = C - 10{,}000$ v.u. $+ W$.

$$\begin{aligned} C - 100.00 \text{ sq. in.} &= 116.91 \\ W &= \phantom{0}41.64 \\ \hline A &= 158.55 \text{ sq. in.} \end{aligned}$$

*Example of Case 4.* With the same set-up on a smaller area the dial moved a little over 1 revolution backward. The reading was 92.35. $A = C - 20{,}000$ v.u. $+ W$.

$$\begin{aligned} C - 200.00 \text{ sq. in.} &= \phantom{0}16.91 \\ W &= \phantom{0}92.35 \\ \hline A &= 109.26 \text{ sq. in.} \end{aligned}$$

## IV. Accuracy

As is the case with all scientific instruments, the accuracy of planimeter observations depends upon several factors. Four of these are discussed below:

1. The precision of the instrument.
2. The conditions affecting the measurement.
3. The human equation.
4. The size of the figure measured.



**1. The Instrument Condition** The precision and accuracy of the instrument are easily determined with the aid of the Test Rule supplied for the purpose. The function of the test rule is to permit the tracer point to accurately circumscribe a circle of known area. To make a test with the test rule, proceed as follows:

(*a*) Draw a fine, straight line on smooth clean paper about 5 in. long, and about 45° below the horizontal. Set up the instrument and test rule as shown in Fig. 13a (The tracer point should be in the center of circumscribed area when pole arm and tracer arm form an angle of approximately 90°).

The movement of the measuring wheel is at a minimum when the tracer is about 45° below or above the horizontal line. Slight errors in the starting or finishing position are therefore minimized.

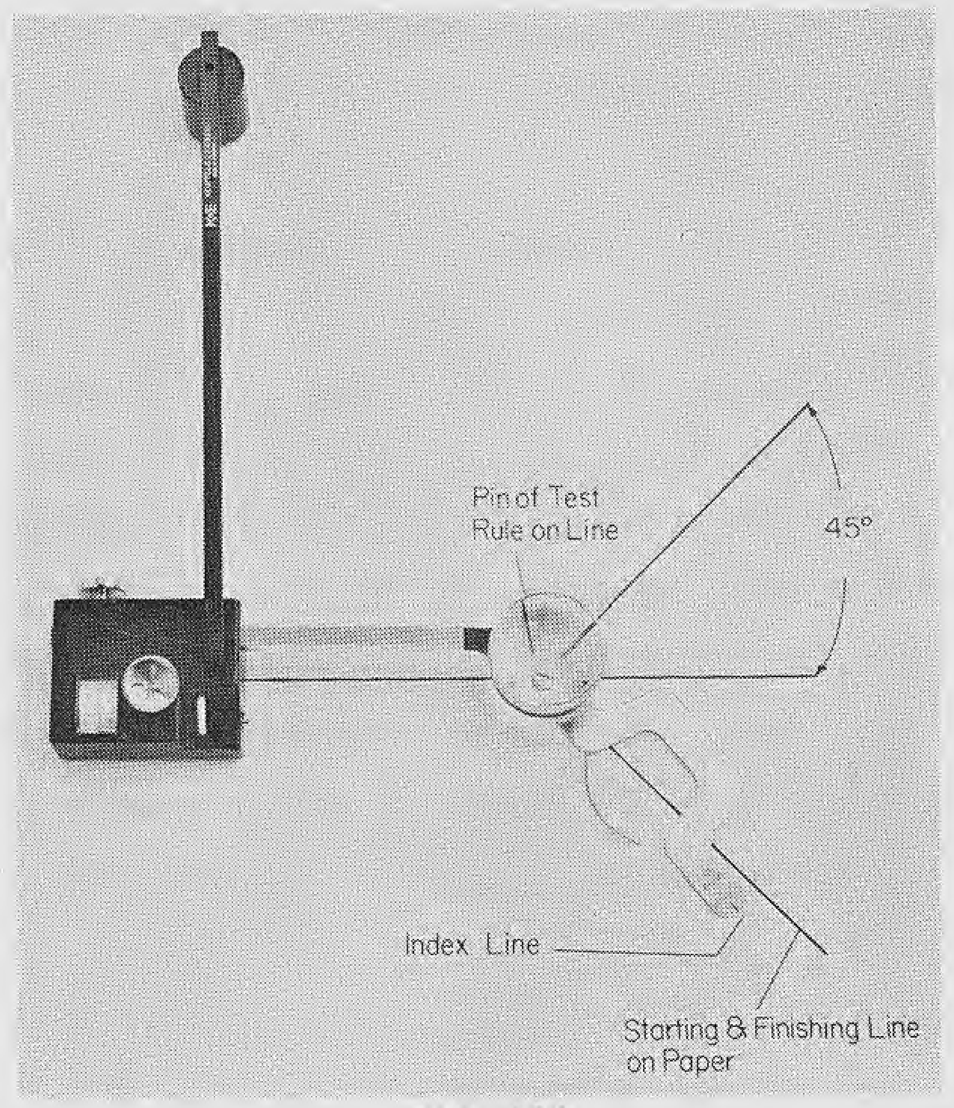



FIG. 13A

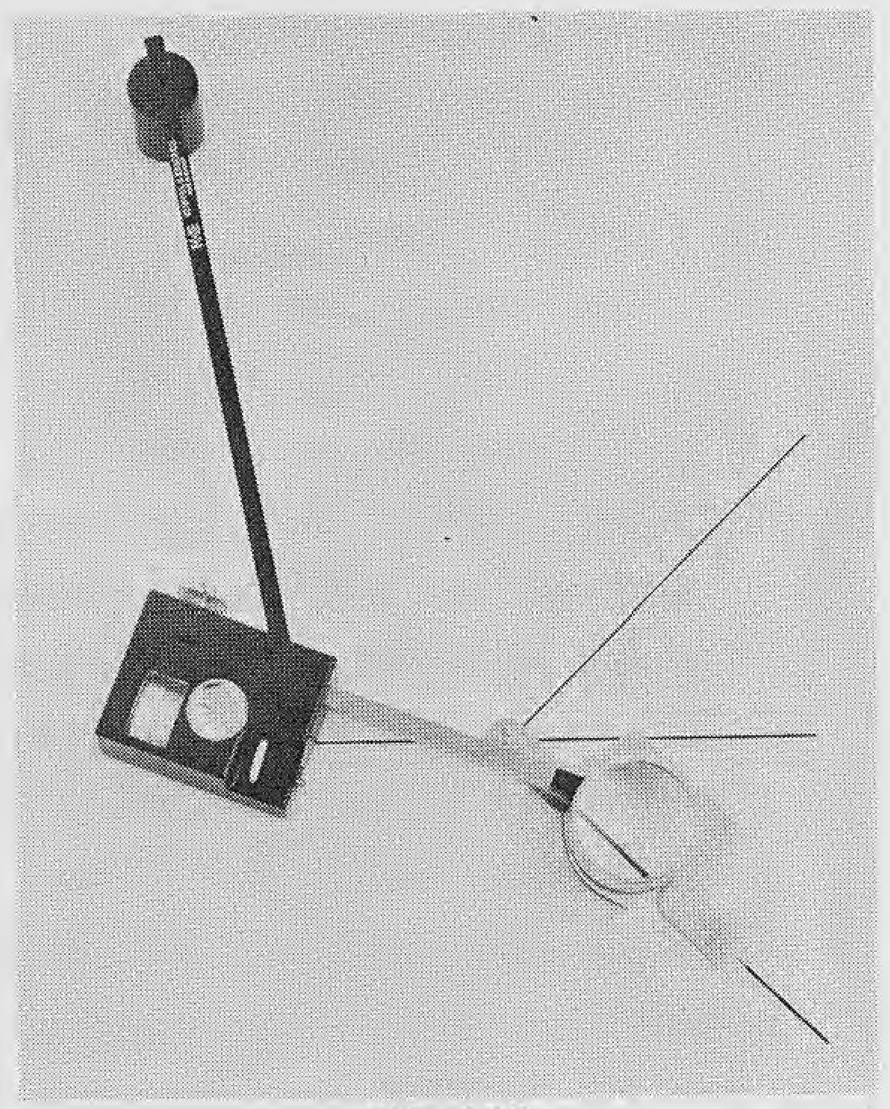


FIG. 13B

(*b*) Place tracer lens in position against test rule and bring index line of the rule precisely to the line on the paper as shown in Fig. 13b*. Set the wheel and dial exactly to 0 ( page 15). Be sure the tracer lens is in contact with the test rule.

(*c*) With index finger, move the test rule clockwise at a moderate uniform speed around a complete circle to the starting point. The tracer lens should at all times be in contact with the test rule during the complete trace. When the index line on the test rule is brought exactly to the line on the paper, the wheel should have made one complete revolution (0 to 0) while the dial has moved from 0 to 1. Record the reading.

(*d*) Make nine more circuits and record the reading after each. Two

**On Planimeters with a tracer point and stop, the stop is raised and the tracer pin is placed in the prick punch depression in the test rule. The tracer pin is held firmly, and the test rule rotated by means of the tracer pin. All other steps are the same as above.*

characteristic sets of readings are given below.

| Test A | Test B |
|---|---|
| 10.02 | 10.00 |
| 20.02 | 20.00 |
| 30.02 | 29.99 |
| 40.02 | 39.99 |
| 50.01 | 49.99 |
| 60.01 | 59.98 |
| 70.02 | 69.97 |
| 80.02 | 79.97 |
| 90.03 | 89.96 |
| 100.03 | 99.96 |

Test A indicates that there was a small error in the initial setting, and that the readings show the planimeter to be accurate to the limit of the operator's ability to read the vernier. Test B shows that there is a slight but definite tendency for the instrument to read low, the apparent error amounting to about 1 part in 2500. Both tests show that the instrument has a precision of better than 1:1000 in measuring a 10 sq. in. area.

(*e*) Further tests may be made by (a) decreasing or increasing the distance between the pole and the test rule, and (b) reversing the pole so that it is at the left instead of at the right. Such tests will show slight variations in results, varying in different instruments. Their principal value is in determining whether the instrument is in good general condition and capable of producing consistent results. If it is found that the instrument reads high when the pole is to the right and low when it is to the left, or vice versa, an equal number of readings may be taken in each position and averaged. This is the "compensating" feature which gives these instruments their name.

**2. Conditions of Measurement** As important as the inherent precision are the conditions external to the instrument. Perhaps the most vital is the surface upon which the measuring wheel rides. If the paper has been folded or torn or has pin-pricks or is wavy or is uneven in any way, accurate results cannot be expected.

A difference in the texture of the paper, map or photograph may cause the instrument to read consistently either somewhat high or low. Check the instrument on the map or photograph with the test rule and any deviation may be corrected by lengthening or shortening the tracer arm. To do this, loosen the two tracer arm set screws on the underside of the carriage. Lengthen the arm slightly if the instrument reads high, shorten the arm if it reads low, until a reading with the test rule gives the exact number of vernier units per revolution. Do not touch the two painted set screws on the back side of the carriage.

Working on a sloping board decreases accuracy somewhat and increases the danger of dropping the instrument on the floor—a serious hazard. Temperature has some effect. Instruments are calibrated to read correctly at 70°F.



**3. The Human Equation** Some operators can obtain much better results than others. Good eyesight, a steady hand and patience in carefully following the outline are important assets. Good light and comfortable working position are also important for accurate work.

Repeated measurements of the same area increase the accuracy of the result, and if tests are made by different operators, their relative efficiency can be judged.

Two sets of readings are given below, both made on the same figure with the same No. 62 0000 planimeter but by different operators.

READINGS BY OPERATOR A

| | *Readings* | | *Areas* | *Deviations* |
|---|---|---|---|---|
| | 3582 | 4874 | 12.92 | +.042 |
| | | 6162 | 12.88 | +.002 |
| | | 7454 | 12.92 | +.042 |
| | | 8737 | 12.83 | −.048 |
| | | 0023 | 12.86 | −.018 |
| | | 1307 | 12.84 | −.038 |
| | | 2597 | 12.90 | +.022 |
| | | 3883 | 12.86 | −.018 |
| | | 5170 | 12.87 | −.008 |
| | | 6460 | 12.90 | +.022 |
| Total | 3582 | 16460 | 128.78 | .260 (Regardless of sign) |
| Average | | | 12.878 sq. in. | **.0260** |

READINGS BY OPERATOR B

| | *Readings* | | *Areas* | *Deviations* |
|---|---|---|---|---|
| | 5784 | 7070 | 12.86 | −.026 |
| | | 8361 | 12.91 | +.024 |
| | | 9650 | 12.89 | +.004 |
| | | 0939 | 12.89 | +.004 |
| | | 2228 | 12.89 | +.004 |
| | | 3517 | 12.89 | +.004 |
| | | 4804 | 12.87 | −.016 |
| | | 6090 | 12.86 | −.026 |
| | | 7380 | 12.90 | +.014 |
| | | 8670 | 12.90 | +.014 |
| Total | 5784 | 18670 | 128.86 | .136 (Regardless of sign) |
| Average | | | 12.886 | **.0136** |

It will be noted that the average deviation in the first set is almost double that of the second, indicating that B is the better operator. Also note that the final results of the two sets agreed within 1 part in 1500, although individual maximum and minimum readings in the first set varied as much as 1:150 and in the second set 1:400. These tests probably represent a fair measure of the accuracy which can be achieved by different operators under favorable conditions on an area of the size indicated.

**4. Accuracy and Size of Figure** Disregarding instrumental errors, if a 1 in. square is traced with the tracer 0.01 in. outside the outline, the area would measure 1.02 $\times$ 1.02 = 1.04 sq. in. approx. If a square 3.16 on a side were similarly measured, the result would be 10.127 instead of 10.00; for a 10.00 in. square, 100.40 instead of 100.00. The percent error drops from 4% to 1.3% to 0.4% successively, and for square figures varies very nearly inversely as the square root of the area. Thus if the average accuracy for a square figure of 10 sq. in. is 1:1000, that of a similar figure 1 sq. in. would be only 1:316, while that of a figure of 100 sq. in. would be 1:3160.

The accuracy to be expected in the measurement of large areas "pole-within-figure" is indicated in Table III showing freehand measurements of large circles. Note that the average deviation is very low and is independent of size. The measured areas are in excellent agreement with the computed areas, the error apparently being due as much to the delineation, diameter measurements and tracing as it is to inherent instrumental errors, since there is no systematic departure from the mean. It is also important to note that there is no impairment of accuracy near the neutral circle, as has been assumed in all previous planimeter instructions.

## Table III

### ACTUAL FREEHAND MEASUREMENT OF CIRCLES POLE-WITHIN-FIGURE

| *1. Nominal diameter* | 8″ | 10″ | 12″ | 14″ | 16″ | 18″ | 20″ | 22″ |
|---|---|---|---|---|---|---|---|---|
| *2. Planimeter readings, v.u.* | 3296 | 6130 | 9595 | 3776 | 8455 | 3903 | 9871 | 6419 |
| *3.* ” | 3319 | 6138 | 9600 | 3789 | 8472 | 3845 | 9855 | 6415 |
| *4.* ” | 3334 | 6155 | 9630 | 3803 | 8494 | 3870 | 9860 | 6410 |
| *5.* ” | 3327 | 6135 | 9626 | 3804 | 8468 | 3853 | 9858 | 6403 |
| *6. Average* | 3319 | 6139 | 9613 | 3793 | 8472 | 3868 | 9861 | 6412 |
| *7. Neutral Circle** | 1691 | 1691 | 1691 | 11691 | 11691 | 21691 | 21691 | 31691 |
| *8. Indicated area sq. in.* | 50.10 | 78.30 | 113.04 | 154.84 | 201.63 | 255.59 | 315.52 | 381.03 |
| *9. Measured diameter in.* | 8.00 | 9.98 | 11.98 | 14.03 | 16.00 | 18.04 | 20.03 | 22.00 |
| *10. Computed area* $\frac{\pi}{4}D^2$ | 50.27 | 78.23 | 112.72 | 154.60 | 201.06 | 255.60 | 315.10 | 380.13 |
| *11. Error in %* | −0.34 | +0.01 | +0.28 | +0.16 | +0.28 | 0.00 | +0.13 | +0.24 |
| *12. Average deviation 4 readings v.u.'s* | 11 | 7 | 15 | 10 | 11 | 19 | 5 | 5 |

*(C − 20,000), (C − 10,000), C, or (C + 10,000) see page 15.

We are, therefore, justified in the statement that with repeated careful measurements under favorable conditions an accuracy of 3 in 1000 can be secured with K&E Polar Planimeters, except where the area is less than 10 sq. in.



## V. Features Special to No. 62 0015*

The No. 62 0015 K&E Compensating Polar Planimeter has an *adjustable* tracer arm and an *adjustable* pole arm. In other respects it is like No. 62 0000 or No. 62 0005. When the tracer arm is set to about 20.30 (the exact setting is shown inside the case), the planimeter will read areas "pole-outside-figure" in square inches (1 v.u. = .01 sq. in.), and all the foregoing instructions regarding No. 62 0000 will apply to the operation. Likewise when the tracer arm is set to about 31.50 (exact setting shown inside the case) the planimeter measures in square centimeters (1 v.u. = 0.1 sq. cm.) as No. 62 0005. For all "pole-outside-figure" measurements the length of the pole arm is immaterial. Included in the instrument case are test rules for 10 sq. in. and 100 sq. cm.

Since the general operation of all these instruments is thoroughly explained in the foregoing text, it is only necessary to add here an explanation of the method of adjusting the arms and the results obtained thereby.

**Adjustment of Tracer Arm** When clamp lever L (Fig. 14) is moved to the right, the carriage D is free to slide on the tracer arm E. A "setting" refers to the position of the vernier V on the tracer arm scale S.

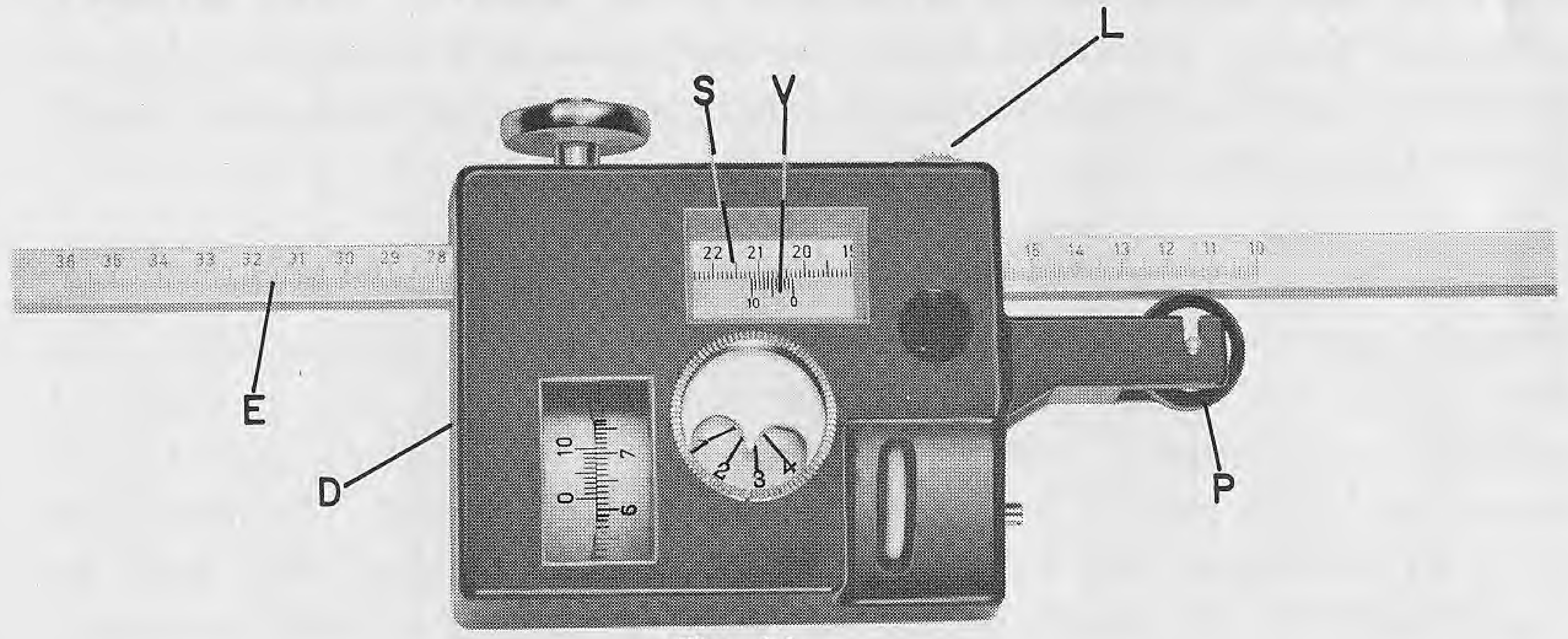



Fig. 14

To make a setting, loosen clamp lever L and slide the carriage so that the vernier zero is near the setting desired. With thumb of right hand press the adjusting wheel P against the tracer arm, and by rotating wheel P gradually move carriage to the exact setting required. Then tighten clamp lever L. For example, to set the carriage to 20.27, loosen clamp lever L, slide carriage so that the vernier zero is about at the second division to the left of 20. With thumb on wheel P move carriage until the vernier zero is between the second and third division to the left of 20, and the seventh division to the left of 0 on the vernier is directly opposite a scale division. Tighten clamp lever. To illustrate,

*Also applies to No. 62 0022.

see setting in Fig. 14. The method of using the vernier is as explained on page 7, but note that in this case the scale and vernier are read from right to left rather than from left to right.

The tracer arm scale is in half-centimeters, so that one vernier unit represents 5 mm. $\times$ 0.01 = 0.05 mm. = 0.002 inches approx. The scale is generally used in a purely arbitrary manner, because each instrument not only has its own particular characteristics, but these may vary slightly with the passage of time, due to wear and minor accidents. All such variations are automatically compensated for by the proper use of the test rule, as explained below.

The length of the pole arm when making "pole-outside-figure" measurements has no effect on the measurement. However, to assure smooth operation and maximum area coverage, the pole arm should be lengthened, if necessary, so that it is at least three inches longer than the tracer arm.

**Measurements to Scale *("Pole-outside-figure")*** As already explained, if readings are made in square inches or square centimeters, the setting of the tracer arm is as given inside the case. With one of these settings, areas can be measured to any scale by using an easily computed factor, as on the fixed arm planimeters. See page 11. If it is desired to eliminate this factor, it can be done by adjusting the length of the tracer arm.

Table IV gives the settings for all commonly used scales in the English system. Note that the tracer arm setting is approximate, but the number of vernier units for use with the testing rule is exact. A column is provided in which the exact tracer arm setting may be put down for future reference after it has been found by test. The procedure is to set the tracer arm to the approximate setting given, then note the exact number of vernier units for one revolution of the test rule. Make one revolution with the 10 sq. in. test rule. Vary the tracer arm setting with adjusting wheel P until the v.u. corresponds exactly to the number given in the table. Lengthening the arm decreases the v.u. per revolution and vice versa. The planimeter will then give readings on the drawing or map, each v.u. having the area value given in the table.

Table V gives the settings to use when the metric system is employed. Here the unit of measurement is 1 sq. cm. and the 100 sq. cm. test rule is used as a standard. For very short tracer arm settings (below 14) the 10 sq. in. test rule must be used, and the reading must be multiplied by the factor 1.55, to give the reading that would result if 100 sq. cm. were used.



## Table IV
### TRACER ARM SETTINGS FOR SCALE DRAWINGS OR MAPS
### No. 62 0015 & No. 62 0022—English System.

| *Nominal Scale of Dwg. or Map* | *Actual Scale Ratio* | *Tracer Arm Setting for 10 sq. in.* | | *Exact v.u.'s per 10 sq. in on Test Rule* | *Value of 1 v.u. to scale* |
|---|---|---|---|---|---|
| | | *Approx.* | *Exact* | | |
| Double size | 2:1 | 16.3 | | 1250 | 0.002 sq. in. |
| $1\frac{1}{2}$ size | $1\frac{1}{2}$:1 | 18.3 | | 1111 | 0.004 " " |
| Full size | 1:1 | 20.3 | | 1000 | 0.01 " " |
| Half size | 1:2 | 20.3 | | 1000 | 0.04 " " |
| Quarter size | 1:4 | 12.7 | | 1600 | 0.1 " " |
| 3 in. = 1 ft. | 1:4 | 18.2 | | 1111 | 0.001 sq. ft. |
| $1\frac{1}{2}$ in. = 1 ft. | 1:8 | 22.8 | | 889 | 0.005 " " |
| 1 in. = 1 ft. | 1:12 | 20.3 | | 1000 | 0.01 " " |
| $\frac{3}{4}$ in. = 1 ft. | 1:16 | 22.8 | | 889 | 0.02 " " |
| $\frac{1}{2}$ in. = 1 ft. | 1:24 | 20.3 | | 1000 | 0.04 " " |
| $\frac{1}{4}$ in. = 1 ft. | 1:48 | 12.7 | | 1600 | 0.1 " " |
| $\frac{1}{8}$ in. = 1 ft. | 1:96 | 31.7 | | 640 | 1.0 " " |
| 1 in. = 10 ft. | 1:120 | 20.3 | | 1000 | 1.0 " " |
| 1 in. = 20 ft. | 1:240 | 20.3 | | 1000 | 4.0 " " |
| 1 in. = 40 ft. | 1:480 | 12.7 | | 1600 | 10.0 " " |
| 1 in. = 50 ft. | 1:600 | 16.3 | | 1250 | 20.0 " " |
| 1 in. = 66 ft. | 1:792 | 18.6 | | 1089 | 40.0 " " |
| 1 in. = 66 ft. | 1:792 | 20.3 | | 1000 | 0.001 acre |
| 1 in. = 80 ft. | 1:960 | 31.7 | | 640 | 100. sq. ft. |
| 1 in. = 100 ft. | 1:1200 | 20.3 | | 1000 | 100. " " |
| 1 in. = 132 ft. | 1:1584 | 11.6 | | 1742 | 100. " " |
| 1 in. = 132 ft. | 1:1584 | 20.3 | | 1000 | 0.004 acre |
| 1 in. = 200 ft. | 1:2400 | 22.1 | | 918 | 0.01 " |
| 1 in. = 400 ft. | 1:4800 | 22.1 | | 918 | 0.04 " |
| 1 in. = $\frac{1}{8}$ mi. | 1:7920 | 20.3 | | 1000 | 0.1 " |
| 1 in. = $\frac{1}{4}$ mi. | 1:15,840 | 20.3 | | 1000 | 0.4 " |
| 1 in. = $\frac{1}{2}$ mi. | 1:31,680 | 25.4 | | 800 | 2.0 " |
| 1 in. = 1 mi. | 1:63,360 | 31.7 | | 640 | 10.0 " |
| — | 1:25,000 | 20.4 | | 996 | 1.0 " |
| — | 1:25,000 | 13.0 | | 1557 | 0.001 sq. mi. |
| — | 1:62,500 | 32.6 | | 623 | 10.0 acre |
| — | 1:62,500 | 20.9 | | 973 | 0.01 sq. mi. |
| — | 1:63,360 | 20.3 | | 1000 | 0.01 " " |

## Table V
### TRACER ARM SETTINGS FOR SCALE DRAWINGS OR MAPS
### No. 62 0015 & No. 62 0022—Metric System.

| *Scale Ratio of Drawing or Map* | *Tracer Arm Setting for 100 sq. cm.* *Approx.* | *Exact* | *Exact v.u.'s per 100 sq. cm. on Test Rule* | *Value of 1 v.u. to scale* |
|---|---|---|---|---|
| 1:1 | 12.6 | | 1613* | 0.04 sq. cm. |
| 1:1 | 31.5 | | 1000 | 0.1 " " |
| 1:1.25 | 20.2 | | 1562 | 0.1 " " |
| 1:1.5 | 13.7 | | 1451* | 0.1 " " |
| 1:2 | 31.5 | | 1000 | 0.4 " " |
| 1:3 | 34.8 | | 900 | 1.0 " " |
| 1:4 | 19.5 | | 1600 | 1.0 " " |
| 1:5 | 25.0 | | 1250 | 2.0 " " |
| 1:6 | 17.3 | | 1800 | 2.0 " " |
| 1:8 | 19.5 | | 1600 | 4.0 " " |
| 1:10 | 31.5 | | 1000 | 10.0 " " |

**Using 10 sq. in. test rule.*

*For ratios which are powers of 10 of the above, use the same setting, but change the value of 1 v.u. accordingly. For 1:200, use setting for 1:2, and 1 v.u. = 0.4 × 100 × 100 = 4,000 sq. cm. = 0.4 sq. m.*

If the drawing or map scale is not found in the tables, the first step is to find the scale factor as described on page 11. In the English system the factor is the area of 1 sq. in. on the map, in the units to be used. If this factor, after multiplying it by an integral power of 10 (that is, by moving its decimal point) is in the range of 565 to 2032 a setting of the tracer arm can be made in which areas can be read directly.

The factor (between 565 and 2032) represents the number of vernier units that the measuring wheel will turn for one revolution of the testing rule (10 sq. in.) when the tracer arm is set to the proper length. To determine quickly the approximate setting, use the graph Fig. 15. For example, to find the setting for a scale of 1:3 in inches: the factor is $3 \times 3 = 9$. As $9 \times 100 = 900$ is in the specified range, follow the vertical line numbered 900 to its intersection with the diagonal marked "square inches" and across to read 22.6 approx. at left or right. Set the tracer arm scale to 22.6 and, using the testing rule (10 sq. in.), adjust wheel P until 1 revolution = 900 v.u. precisely. The instrument is now set to read areas directly to the desired scale. Thus with this setting, if a measured area reads 2756 v.u., the area represented on the drawing is 275.6 sq. in. The value of one vernier unit is equal to the factor times the area of the test rule, divided by the vernier units per one revolution of the test rule. In the above example:

$$1 \text{ v.u.} = \frac{9 \times 10}{900} = .1 \text{ sq. in.}$$

When the metric system is used, the same principles apply, but in this



## GRAPH FOR FINDING TRACER ARM SETTINGS FOR SCALE DRAWINGS & MAPS

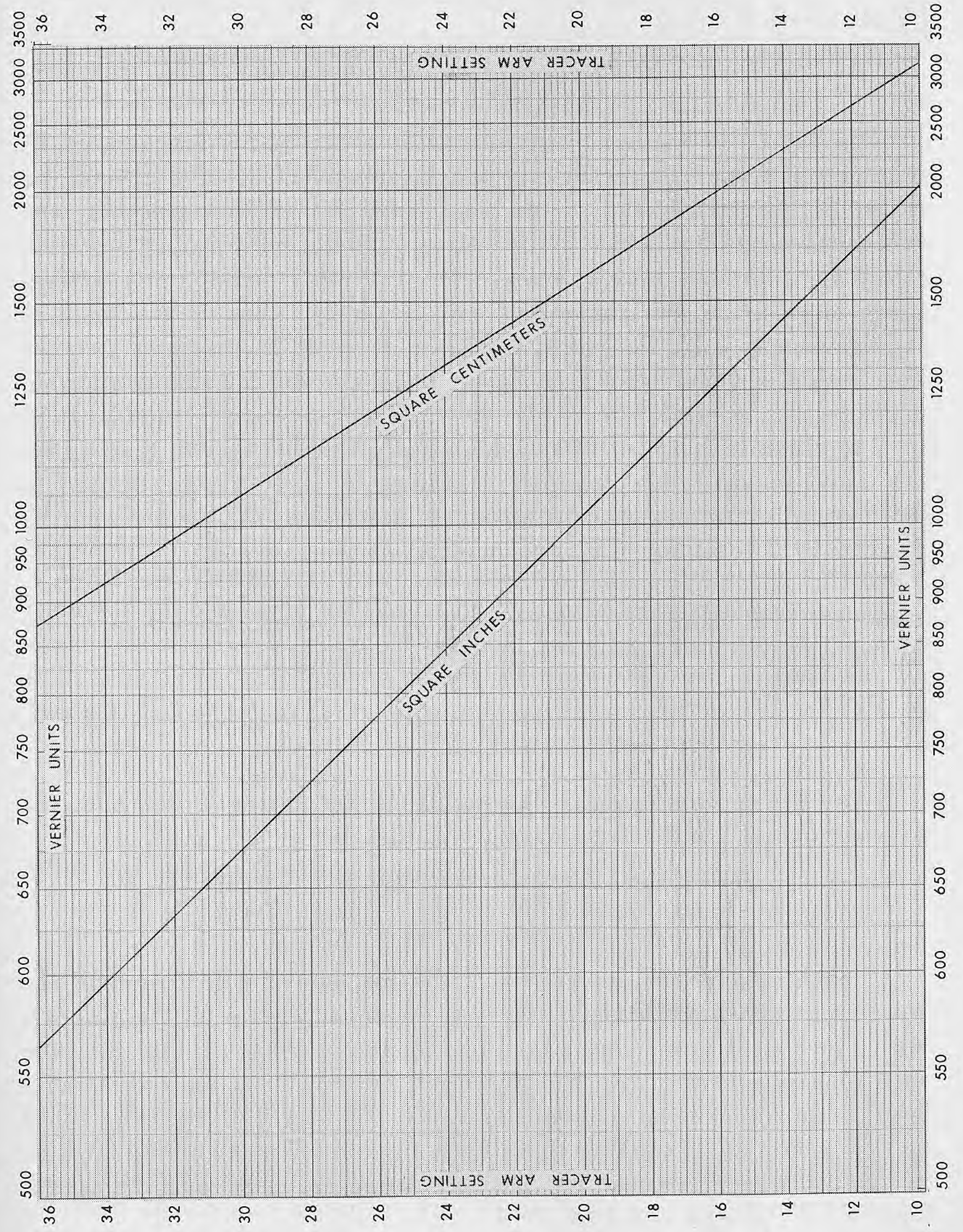



Fig. 15

case the range of factors is from 875 to 3150. The curve marked "square centimeters" on Fig. 15 indicates the tracer arm setting, and the 100 sq. cm. test rule is used to establish the exact setting.

In the cases where the factor cannot be brought within the range for direct reading it is possible to divide the factor by a convenient number such as 2, 3 or 4, and then multiply the planimeter reading by the same number. For example, if the scale is 1:2.2 the factor is $2.2 \times 2.2 = 4.84$. Multiply by 1000 and divide 4840 by 4 to obtain 1210. By the above formula:

$$1 \text{ v.u.} = \frac{4.84 \times 100}{1210} = .4 \text{ sq. cm.}$$

This method of manipulating the factor as desired is often useful in obtaining a tracer arm setting which is most convenient; that is, near the center or near either end of the arm. Settings near the center are the best for average use, while short settings give the most accurate results, but with a limited area coverage. On the other hand, long settings cover large areas, but somewhat less accurately. Thus, if the scale of the drawing is 1:14, the factor would be 1960 which indicates a tracer arm setting of about 10.4. This setting could be used but the measurable areas would be small. To obtain a good average setting, divide 1960 by 2 to obtain a factor of 980, which gives a tracer arm setting of about 20.7, and multiply the planimeter readings by 2. If it were desired to cover large areas and sacrifice accuracy somewhat, the factor could be divided by 3 to obtain 653, and multiply readings by 3.

**Indicator Diagrams** The instrument is so constructed that $\pi D = 2.5$ in. in the formula $A = l\pi DN$ (page 31). Therefore

$$\frac{A}{l} = 2.5N$$

If the tracer arm is set to the length of the indicator diagram, the average height *(h)* of the diagram will then be

$$h = \frac{A}{l} = 2.5N \qquad \text{or} \qquad N = 0.4\ h$$

As each square inch is 1000 v.u., the instrument will then read the average height on the basis of 400 v.u. per inch. Square off the diagram with fine pencil lines. (See Fig. 11.) Set the carriage so that the pole arm hole is centered over the lower left corner. Then slide the tracer arm to the right corner, so that the tracer arm length, from tracer point to pole arm hole, is equal to that of the diagram. Tighten clamp lever L. With this tracer arm setting, trace the diagram and read the wheel. The M.E.P. is then equal to

$$\frac{\text{area in v.u.} \times \text{pressure scale}}{400}$$



*Example.* The pressure scale is 80 lb./in.$^2$ per inch. The traced area read 355 v.u. The M.E.P. is $\frac{355}{400} \times 80 = 71.0$ lb./in.$^2$

**Adjustment of Pole Arm** The length of the pole arm is immaterial on "pole-outside-figure" measurements. Therefore, where space is limited, the pole arm may be shortened; or, where areas are large, a long pole arm will be found convenient.

In "pole-within-figure" measurements, the pole arm length is a vital factor in the result, as explained in the section on the theory (page 30).

The pole arm setting given on the lid of the case (about 31.0) is the one which produces a neutral circle area of 200.00 sq. in. when the tracer arm is set for reading in square inches (about 20.3). With this setting, the four formulas on page 15 become:

(1) $A = W + 300.00$ sq. in.
(2) $A = W + 200.00$ sq. in.
(3) $A = W + 100.00$ sq. in.
(4) $A = W$

where $A$ is the area in square inches and $W$ is the planimeter reading in hundreds of v.u.'s.

When the metric system is used and the tracer arm is set for metric measure (about 31.5) as shown on the label, the area of the neutral circle will be 2000.0 sq. cm. if the pole arm is set as follows: move the pole arm index 2.94 half-cm. above the pole arm setting given on the box lid. For example, if the pole arm setting on the box lid is given as 30.72, the setting for 2000 sq. cm. neutral circle will be

$$30.72 + 2.94 = 33.66$$

With this setting the four formulas on page 15 become:

(1) $A = W + 3000.0$ sq. cm.
(2) $A = W + 2000.0$ sq. cm.
(3) $A = W + 1000.0$ sq. cm.
(4) $A = W$

where $A$ is the area in sq. cm. and $W$ is the planimeter reading in tens of v.u.'s.

If it is desired to measure extra large areas by increasing the length of the pole arm and hence the area of the neutral circle, the table below will be found useful. The pole arm length is set with the aid of an accurate measuring scale, the length being the distance between the center of the ball and the pole needle.

| *Tracer Arm Setting** | *Length of Pole Arm* | *Area of Neutral Circle* |
|---|---|---|
| About 20.3 | 8.37 in. | 300.00 sq. in. |
| " " | 10.11 " | 400.00 " " |
| " " | 11.57 " | 500.00 " " |
| About 31.5 | 24.80 cm. | 3000.0 sq. cm. |
| " " | 30.55 " | 4000.0 " " |

In making measurements using any of these settings the formulas for the four cases apply, namely

Case 1. $A = C + 10{,}000 \text{ v.u.} + W$
Case 2. $A = C + W$
Case 3. $A = C - 10{,}000 \text{ v.u.} + W$
Case 4. $A = C - 20{,}000 \text{ v.u.} + W$

Substitute the known value of $C$ (area of neutral circle in v.u.), and convert v.u.'s to the appropriate unit of area. For example, if the pole arm is set to 10.11 in., $C = 400$ sq. in., and 10,000 v.u. = 100 sq. in., the formulas become

$$A = 400 + 100 + W = W + 500 \text{ sq. in.}$$
$$A = 400 + W = W + 400 \text{ sq. in.}$$
$$A = 400 - 100 + W = W + 300 \text{ sq. in.}$$
$$A = 400 - 200 + W = W + 200 \text{ sq. in.}$$

**Measurements to Scale (*"Pole-within-figure"*)** When large areas are to be measured to scale on scale drawings or maps, two optional procedures are available:

1. Measure the area in square inches or square centimeters and convert to the scale area by multiplying by a factor as explained on pages 11, 12 and 13.

   *Example:* Map scale 1 in. = 200 ft.; wanted measured area in acres. Referring to Table I, 1 sq. in. = 0.918 acres. The tracer arm and pole arm were set to the constants on the box lid, the area was traced and found to read 220.0 sq. in. The area is 220.0 × 0.918 = 202.0 acres.

   This method is recommended because of its simplicity and freedom from potential error. The one computation necessary is easily performed on the slide rule which may be left set at the factor for repeated use without resetting.

2. Measure the area to scale. With this method the tracer arm may be set as indicated in Table IV or V but the pole arm setting must be determined by computation, using the formula for the neutral circle (see theory, page 30) arranged to determine $r$, the other factors being known:

$$r = \sqrt{\frac{c}{\pi} - l^2 - 2al} \quad \text{where}$$

*As per chart in lid of case.



$r$ is the pole arm length in inches (or cm.)
$c$ is the neutral circle area in sq. in. (or sq. cm.)
$l$ is the tracer arm length in inches (or cm.)
$a$ is equal to 1.199 inches (or 3.045 cm.) for No. 62 0015.

*Example:* Map scale 1 in. = 200 ft.; wanted measured area in acres. The tracer arm setting (Table IV) is found by test to be 22.16 half-cm. = 4.36 in. Let the neutral circle area = 300 acres = 300 ÷ 0.918 = 327 sq. in.

$$r = \sqrt{\frac{327}{\pi} - 4.36^2 - 2 \times 1.199 \times 4.36} = 8.63 \text{ in.}$$

With an accurate scale, set the distance on the pole arm between the center of the ball and the pole needle to 8.63 inches. Areas measured with these settings will be in acres, based on a neutral circle area of 300 acres.

A check on the exact area of the neutral circle for any tracer arm and pole arm setting whatever can be made as follows:

1. With a good drawing compass draw a circle of exactly 11.284 inches in diameter. This circle has an area of 100.00 sq. in.
2. Placing the pole near the center of the circle carefully trace the area several times and find the average reading $W$ in sq. in., noting also how many complete negative turns of the dial were observed, and apply the proper case:

   Case 3. Less than 1 complete turn.
   Case 4. Between 1 and 2 complete turns.
   Case 5. " 2 " 3 " "
   Case 6. " 3 " 4 " "

3. The area $C$ of the neutral circle is:

   Case 3. $C$ sq. in. = 200.00 sq. in. − $W$ sq. in.
   Case 4. $C$ " " = 300.00 " " − $W$ " "
   Case 5. $C$ " " = 400.00 " " − $W$ " "
   Case 6. $C$ " " = 500.00 " " − $W$ " "

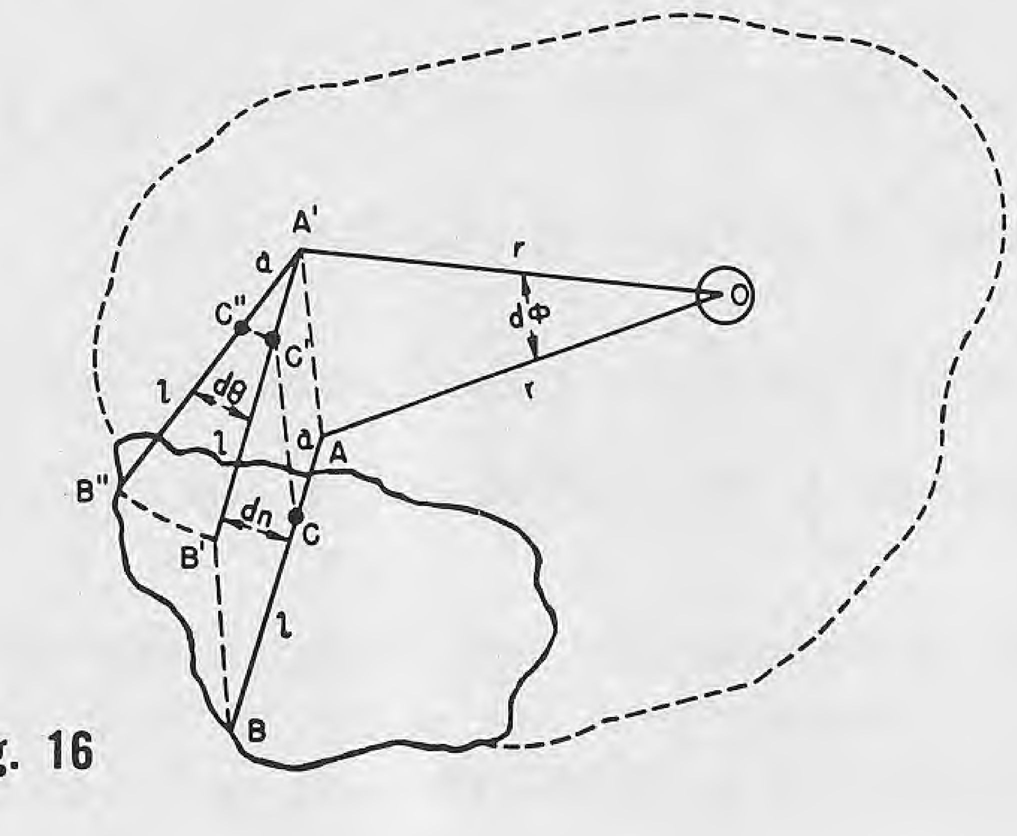



Fig. 16

## VI. General Theory of the Polar Planimeter

Although algebraic solutions of the planimeter problem have been worked out, they are very involved, whereas the solution using calculus is quite simple, and can be understood by anyone familiar with the rudiments of that subject.

In Fig. 16, $O$ is the pole, $A$, the ball-and-socket joint, $B$ the tracer point, and $C$ the contact point of the measuring wheel; $r$ and $l$ are the lengths of pole arm and tracer arm respectively, and $a = CA$.

Call $dA$, the infinitesimal area $OABB'B''A'O$, swept out by $r$ and $l$ as the tracer arm moved, first from $AB$ to $A'B'$ parallel to itself, and then swung to $A'B''$ around $A'$. Call $\angle AOA' = d\phi$, $\angle B'A'B'' = d\theta$, and the perpendicular distance between $AB$ and $A'B' = dn$; then

$$dA = l\,dn + \tfrac{1}{2}\,l^2\,d\theta + \tfrac{1}{2}\,r^2\,d\phi \qquad (1)$$

As the wheel moved from $C$ to $C'$, it advanced $dn$, since it can be thought of as rolling perpendicular to $AB$ and then sliding to $C'$. From there its motion was all rolling and was $a d\theta$. Thus $dS$, the total linear motion of the wheel rim, was

$$dS = dn + a\,d\theta \qquad (2)$$

Substituting in (1) the value of $dn$ in (2).

$$dA = l\,ds - a\,l\,d\theta + \tfrac{1}{2}\,l^2 d\theta + \tfrac{1}{2}\,r^2 d\phi \qquad (3)$$

When the tracer point covers any area such as the solid or dotted irregular outlines in Fig. 16 and returns to the starting point, the motion from point to point may be considered as the succession of such parallel motions and rotary motions of the tracer arm $AB$. Thus, $A$ the total area swept out by $r$ and $l$ is

$$A = \int dA = l\,S + (\tfrac{1}{2}\,l^2 - a\,l) \int d\theta + \tfrac{1}{2}\,r^2 \int d\phi \qquad (4)$$

in which $S$ is the net linear motion of the wheel rim, $\int d\theta$ is the net angular displacement of $AB$ *and* $\int d\phi$ that of $OA$.

If the pole is outside the figure traced, as in the solid irregular area, Fig. 16, the net value of $\int d\theta = 0$ and of $\int d\phi = 0$, so equation (4) reduces to

$$A = lS$$



To convert $S$ from linear to angular measure,

$$S = \pi\,D\,N$$

where $D$ is the wheel diameter and $N$ is the number of revolutions. Thus

$$A = l\,\pi\,D\,N \qquad (5)$$

Since $l$ and $D$ are constant for any planimeter setting, the area traced is directly proportional to the net angular motion of the measuring wheel. Q.E.D.

If the figure traced goes around the pole ("pole-within-figure")—the dotted irregular area, (Fig. 16) the net value of $\int d\theta = 2\pi$ and of $\int d\phi = 2\pi$ and equation (4) becomes

$$A = l\,\pi\,D\,N + (l^2 - 2\,a\,l + r^2)\,\pi \qquad (6)$$

When the ball and socket joint is located between the wheel and the tracer point, a similar demonstration will yield an equation like equation (6) except that the sign of the term $2al$ will be plus.

The last term in (6) represents the area of the neutral circle. To prove this, set the planimeter elements in position for sliding of the wheel $C$ as in Fig. 17. $R$ is the radius of the neutral circle and

$R^2 = BC^2 + OC^2 = (l - a)^2 + (r^2 - a^2) = l^2 - 2\,a\,l + r^2$

Area of neutral circle $= \pi R^2 = \pi\,(l^2 - 2\,al + r^2)$.

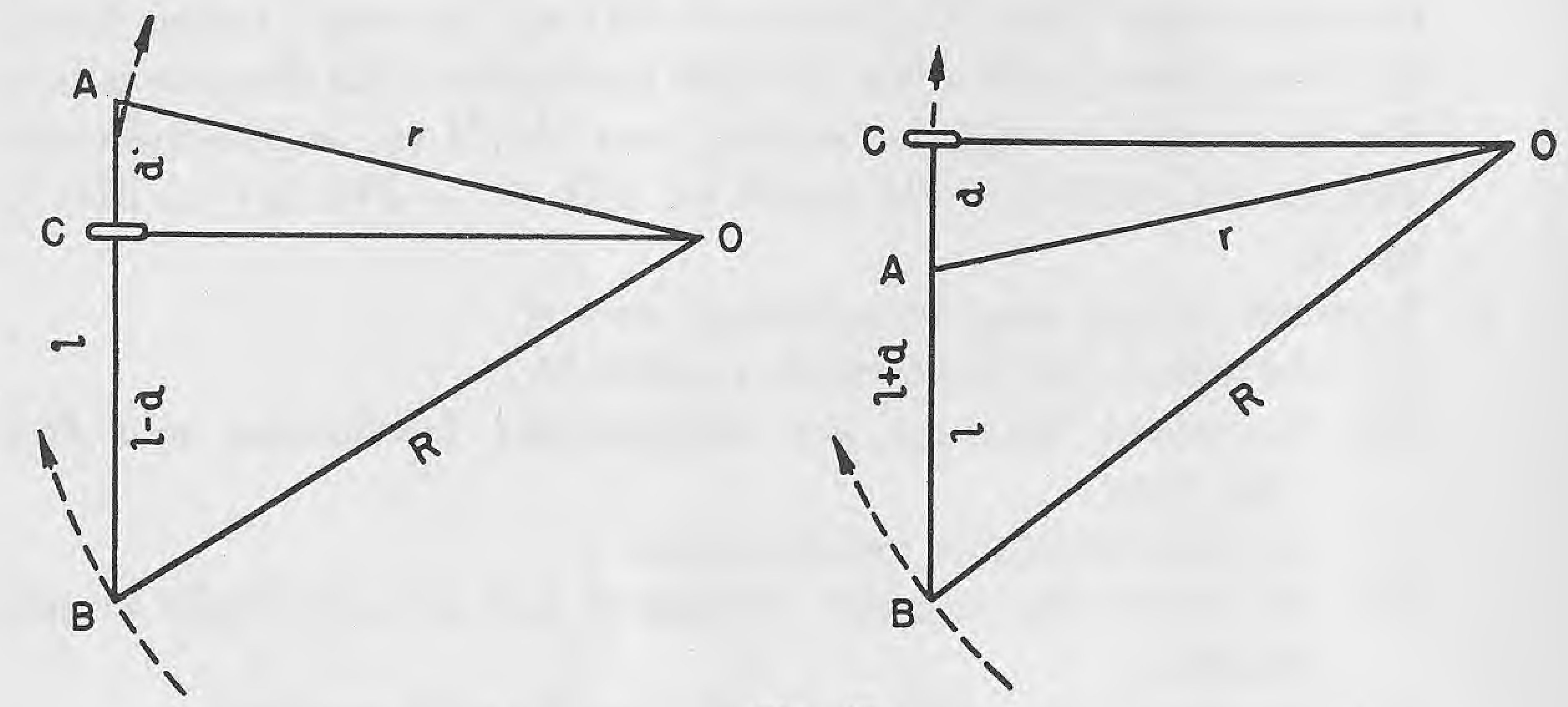



Fig. 17

Thus for "pole-within-figure" the area measured equals the area of the neutral circle plus or minus the area measured on the wheel, depending on whether that area was positive or negative. Q.E.D.

In using planimeters No. 62 0000 and No. 62 0005 the values of $\pi D$ and of $a$ are immaterial as there is never any occasion to use these values since $l$ and $r$ are of fixed length. But on No. 62 0015 it may be desirable to make computations for $l$ and $r$, so the following information is supplied:

$$\pi D = 2.500 \text{ in.} = 6.350 \text{ cm.}$$
$$a = 1.199 \text{ in.} = 3.045 \text{ cm.}$$

The scale on the tracer arm is in half centimeters and measures the length of the tracer arm E (Fig. 14) at the vernier index. The scale on the pole arm is in half centimeters but does not indicate the true length of the pole arm.

From these proofs some interesting facts appear about planimeter design and operation.

1. For "pole-outside-figure" measurements, results are independent of the lengths $r$ and $a$.
2. No appreciable error is introduced by making the axis of the wheel parallel to, rather than directly in $l$, since parallel and angular motion are not directly affected thereby.
3. From equation (5), for a given area, as $l$ increases $N$ decreases proportionally and vice versa. This means that more accurate readings can be made when $l$ is short, since the limit of measurement of $N$ is always 1 vernier unit.
4. To determine the area of the neutral circle, it is not necessary to know the values of $l$, $a$, and $r$, as it can be measured. Select a known value of $A$, measure it, "pole-within-figure", and read. Then the area of the neutral circle equals the known area minus the area read. Thus if a circle of 100 sq. in. area (radius 5.642 in.) were traced with a No. 62 0000 planimeter with the pole within the figure and the negative reading was 116.71 sq. in., the indicated area of the neutral circle would be 100 — (—116.71) = 216.71 sq. in.
5. Accurate results may be expected only if
   (a) the wheel axis is precisely parallel to $l$;
   (b) the wheel bearings are substantially frictionless and free from play;
   (c) the wheel assembly inertia is low;
   (d) the wheel rim is truly concentric and of a precisely known diameter;
   (e) the wheel rim surface can grip the drawing surface;
   (f) the wheel and vernier graduations are uniform and correct.

K&E Planimeters are made with scrupulous regard to these and other important factors essential to accuracy and long service.




10 — 8 — 1/100 DIDGET
0 — 1/10 DIDGET
7 — 1s DIDGET
10s DIDGET
07.24

## Table I
## FACTORS FOR SCALE DRAWINGS OR MAPS
## No. 62 0000, No. 62 0002, No. 62 0015 or No. 62 0022—English System

| Nominal Scale of Dwg. or Map | Actual Scale Ratio | FACTORS — Factors by which sq. in. values must be multiplied to convert to units shown at column headings. sq. in. | sq. ft. | acres | sq. miles |
|---|---|---|---|---|---|
| Double size | 2:1 | 0.25 | — | — | — |
| 1½ size | 1½:1 | 0.444 | — | — | — |
| Full size | 1:1 | 1.0 | — | — | — |
| Centimeters | 1:1 | 6.45* | — | — | — |
| Half size | 1:2 | 4.0 | — | — | — |
| Quarter size | 1:4 | 16. | — | — | — |
| 3 in. = 1 ft. | 1:4 | 16. | 0.111 | — | — |
| 1½ in. = 1 ft. | 1:8 | 64. | 0.444 | — | — |
| 1 in. = 1 ft. | 1:12 | 144. | 1.0 | — | — |
| ¾ in. = 1 ft. | 1:16 | 256. | 1.778 | — | — |
| ½ in. = 1 ft. | 1:24 | 576. | 4.0 | — | — |
| ⅜ in. = 1 ft. | 1:32 | 1024. | 7.11 | — | — |
| ¼ in. = 1 ft. | 1:48 | 2304. | 16.0 | — | — |
| ⅛ in. = 1 ft. | 1:96 | 9216. | 64.0 | — | — |
| 1 in. = 10 ft. 15 | 1:120 | — | 100. 225 | — | — |
| 1 in. = 20 ft. 30 | 1:240 | — | 400. 900 | — | — |
| 1 in. = 40 ft. | 1:480 | — | 1600. | 0.0367 | — |
| 1 in. = 50 ft. | 1:600 | — | 2500. | 0.0574 | — |
| 1 in. = 66 ft. | 1:792 | — | 4356. | 0.1 | — |
| 1 in. = 80 ft. | 1:960 | — | 6400. | 0.147 | — |
| 1 in. = 100 ft. | 1:1200 | — | 10,000. | 0.2296 | — |
| 1 in. = 132 ft. | 1:1584 | — | 17,424. | 0.4 | — |
| 1 in. = 200 ft. | 1:2400 | — | 40,000. | 0.918 | — |
| 1 in. = 330 ft. | 1:3960 | — | 108.900. | 2.5 | — |
| 1 in. = 400 ft. | 1:4800 | — | 160,000. | 3.673 | — |
| 1 in. = 660 ft. | 1:7920 | — | 435,600. | 10.0 | 0.0156 |
| 1 in. = ¼ mi. | 1:15,840 | — | — | 40.0 | 0.0625 |
| 1 in. = ½ mi. | 1:31,680 | — | — | 160.0 | 0.25 |
| 1 in. = 1 mi. | 1:63,360 | — | — | 640.0 | 1.0 |
| — | 1:25,000 | — | — | 99.6 | 0.1557 |
| — | 1:62,500 | — | — | 622.7 | 0.973 |

1/16 = 1 FT 1:192 256

*This factor is square centimeters for reading areas in the metric system.

1/32 - 1 FT 1:384 1024

3/16 - 1 FT 1:64

### No. 62 0010—English System

No. 62 0010 is a fixed arm planimeter for use on maps with scale of 1 in. = 330 ft. It gives direct readings in acres to 0.1 acres. (Reading x 0.1 = area in acres). It can also be used for measuring areas in square inches to 0.04 sq. in. (Reading x 0.04 = area in square inches).